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CHAPTER TWO

UNITS AND MEASUREMENT

2.1 INTRODUCTION

Measurement of any physical quantity involves comparison with a certain basic, arbitrarily chosen, internationally accepted reference standard called **unit**. The result of a measurement of a physical quantity is expressed by a number (or numerical measure) accompanied by a unit. Although the number of physical quantities appears to be very large, we need only a limited number of units for expressing all the physical quantities, since they are inter-related with one another. The units for the fundamental or base quantities are called **fundamental** or **base units**. The units of all other physical quantities can be expressed as combinations of the base units. Such units obtained for the derived quantities are called **derived units**. A complete set of these units, both the base units and derived units, is known as the **system of units**.

2.2 THE INTERNATIONAL SYSTEM OF UNITS

In earlier time scientists of different countries were using different systems of units for measurement. Three such systems, the CGS, the FPS (or British) system and the MKS system were in use extensively till recently.

The base units for length, mass and time in these systems were as follows :

- In CGS system they were centimetre, gram and second respectively.
- In FPS system they were foot, pound and second respectively.
- In MKS system they were metre, kilogram and second respectively.

The system of units which is at present internationally accepted for measurement is the *Système Internationale d' Unites* (French for International System of Units), abbreviated as SI. The SI, with standard scheme of symbols, units and abbreviations, developed by the Bureau International des Poids et measures (The International Bureau of Weights and Measures, BIPM) in 1971 were recently revised by the General Conference on Weights and Measures in November 2018. The scheme is now for international usage in scientific, technical, industrial and commercial work. Because SI units used decimal system, conversions within the system are quite simple and convenient. We shall follow the SI units in this book.

In SI, there are seven base units as given in Table 2.1. Besides the seven base units, there are two more units that are defined for (a) plane angle $d\theta$ as the ratio of length of arc ds to the radius r and (b) solid angle $d\Omega$ as the ratio of the intercepted area dA of the spherical surface, described about the apex O as the centre, to the square of its radius r, as shown in Fig. 2.1(a) and (b) respectively. The unit for plane angle is radian with the symbol rad and the unit for the solid angle is steradian with the symbol sr. Both these are dimensionless quantities.

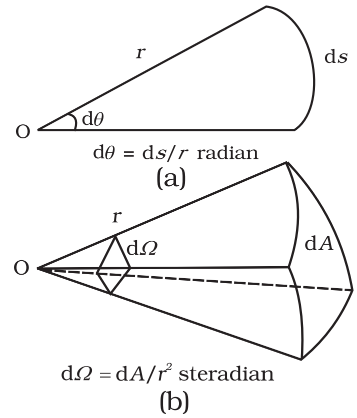

Fig. 2.1 *Description of (a) plane angle $d\theta$ and (b) solid angle $d\Omega$.*

Table 2.1 SI Base Quantities and Units*

Base quantity	SI Units		
	Name	**Symbol**	**Definition**
Length	metre	m	The metre, symbol m, is the SI unit of length. It is defined by taking the fixed numerical value of the speed of light in vacuum c to be 299792458 when expressed in the unit m s^{-1}, where the second is defined in terms of the caesium frequency $\Delta\nu cs$.
Mass	kilogram	kg	The kilogram, symbol kg, is the SI unit of mass. It is defined by taking the fixed numerical value of the Planck constant h to be 6.62607015×10^{-34} when expressed in the unit J s, which is equal to kg m^2 s^{-1}, where the metre and the second are defined in terms of c and $\Delta\nu cs$.
Time	second	s	The second, symbol s, is the SI unit of time. It is defined by taking the fixed numerical value of the caesium frequency $\Delta\nu cs$, the unperturbed ground-state hyperfine transition frequency of the caesium-133 atom, to be 9192631770 when expressed in the unit Hz, which is equal to s^{-1}.
Electric	ampere	A	The ampere, symbol A, is the SI unit of electric current. It is defined by taking the fixed numerical value of the elementary charge e to be $1.602176634\times10^{-19}$ when expressed in the unit C, which is equal to A s, where the second is defined in terms of $\Delta\nu cs$.
Thermo dynamic Temperature	kelvin	K	The kelvin, symbol K, is the SI unit of thermodynamic temperature. It is defined by taking the fixed numerical value of the Boltzmann constant k to be 1.380649×10^{-23} when expressed in the unit J K^{-1}, which is equal to kg m^2 s^{-2} k^{-1}, where the kilogram, metre and second are defined in terms of h, c and $\Delta\nu cs$.
Amount of substance	mole	mol	The mole, symbol mol, is the SI unit of amount of substance. One mole contains exactly 6.02214076×10^{23} elementary entities. This number is the fixed numerical value of the Avogadro constant, N_A, when expressed in the unit mol^{-1} and is called the Avogadro number. The amount of substance, symbol n, of a system is a measure of the number of specified elementary entities. An elementary entity may be an atom, a molecule, an ion, an electron, any other particle or specified group of particles.
Luminous intensity	candela	cd	The candela, symbol cd, is the SI unit of luminous intensity in given direction. It is defined by taking the fixed numerical value of the luminous efficacy of monochromatic radiation of frequency 540×10^{12} Hz, K_{cd}, to be 683 when expressed in the unit lm W^{-1}, which is equal to cd sr W^{-1}, or cd sr $kg^{-1}m^{-2}s^3$, where the kilogram, metre and second are defined in terms of h, c and $\Delta\nu cs$.

* *The values mentioned here need not be remembered or asked in a test. They are given here only to indicate the extent of accuracy to which they are measured. With progress in technology, the measuring techniques get improved leading to measurements with greater precision. The definitions of base units are revised to keep up with this progress.*

Table 2.2 Some units retained for general use (Though outside SI)

Name	Symbol	Value in SI Unit
minute	min	60 s
hour	h	60 min = 3600 s
day	d	24 h = 86400 s
year	y	365.25 d = 3.156×10^7 s
degree	°	$1° = (\pi/180)$ rad
litre	L	I $dm^3 = 10^{-3}$ m^3
tonne	t	10^3 kg
carat	c	200 mg
bar	bar	0.1 MPa = 10^5 Pa
curie	Ci	3.7×10^{10} s^{-1}
roentgen	R	2.58×10^{-4} C/kg
quintal	q	100 kg
barn	b	100 $fm^2 = 10^{-28}$ m^2
are	a	1 $dam^2 = 10^2$ m^2
hectare	ha	1 $hm^2 = 10^4$ m^2
standard atmospheric pressure	atm	101325 Pa = 1.013×10^5 Pa

Note that when mole is used, the elementary entities must be specified. These entities may be atoms, molecules, ions, electrons, other particles or specified groups of such particles.

We employ units for some physical quantities that can be derived from the seven base units (Appendix A 6). Some derived units in terms of the SI base units are given in (Appendix A 6.1). Some SI derived units are given special names (Appendix A 6.2) and some derived SI units make use of these units with special names and the seven base units (Appendix A 6.3). These are given in Appendix A 6.2 and A 6.3 for your ready reference. Other units retained for general use are given in Table 2.2.

Common SI prefixes and symbols for multiples and sub-multiples are given in Appendix A2. General guidelines for using symbols for physical quantities, chemical elements and nuclides are given in Appendix A7 and those for SI units and some other units are given in Appendix A8 for your guidance and ready reference.

2.3 MEASUREMENT OF LENGTH

You are already familiar with some direct methods for the measurement of length. For example, a metre scale is used for lengths from 10^{-3} m to 10^2 m. A vernier callipers is used for lengths to an accuracy of 10^{-4} m. A screw gauge and a spherometer can be used to measure lengths as less as to 10^{-5} m. To measure lengths beyond these ranges, we make use of some special indirect methods.

2.3.1 Measurement of Large Distances

Large distances such as the distance of a planet or a star from the earth cannot be measured directly with a metre scale. An important method in such cases is the **parallax method**.

When you hold a pencil in front of you against some specific point on the background (a wall) and look at the pencil first through your left eye A (closing the right eye) and then look at the pencil through your right eye B (closing the left eye), you would notice that the position of the pencil seems to change with respect to the point on the wall. This is called **parallax**. The distance between the two points of observation is called the **basis**. In this example, the basis is the distance between the eyes.

To measure the distance D of a far away planet S by the parallax method, we observe it from two different positions (observatories) A and B on the Earth, separated by distance AB = b at the same time as shown in Fig. 2.2. We measure the angle between the two directions along which the planet is viewed at these two points. The ∠ASB in Fig. 2.2 represented by symbol θ is called the **parallax angle** or **parallactic angle**.

As the planet is very far away, $\frac{b}{D} << 1$, and therefore, θ is very small. Then we approximately take AB as an arc of length b of a circle with centre at S and the distance D as the radius AS = BS so that AB = $b = D\theta$ where θ is in radians.

$$D = \frac{b}{\theta} \qquad (2.1)$$

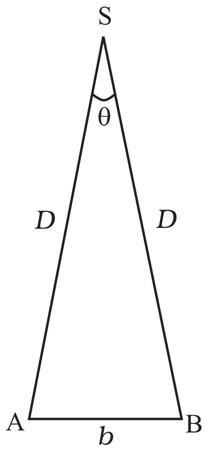

Fig. 2.2 *Parallax method.*

Having determined *D*, we can employ a similar method to determine the size or angular diameter of the planet. If *d* is the diameter of the planet and α the angular size of the planet (the angle subtended by *d* at the earth), we have

$$\alpha = d/D \qquad (2.2)$$

The angle α can be measured from the same location on the earth. It is the angle between the two directions when two diametrically opposite points of the planet are viewed through the telescope. Since *D* is known, the diameter *d* of the planet can be determined using Eq. (2.2).

Example 2.1 Calculate the angle of (a) 1^0 (degree) (b) 1′ (minute of arc or arcmin) and (c) 1″(second of arc or arc second) in radians. Use $360^0=2\pi$ rad, $1^0=60'$ and $1' = 60''$

Answer (a) We have $360^0 = 2\pi$ rad

$1^0 = (\pi/180)$ rad $= 1.745\times10^{-2}$ rad

(b) $1^0 = 60' = 1.745\times10^{-2}$ rad

$1' = 2.908\times10^{-4}$ rad $\simeq 2.91\times10^{-4}$ rad

(c) $1' = 60'' = 2.908\times10^{-4}$ rad

$1'' = 4.847\times10^{-4}$ rad $\simeq 4.85\times10^{-6}$ rad

Example 2.2 A man wishes to estimate the distance of a nearby tower from him. He stands at a point A in front of the tower C and spots a very distant object O in line with AC. He then walks perpendicular to AC up to B, a distance of 100 m, and looks at O and C again. Since O is very distant, the direction BO is practically the same as AO; but he finds the line of sight of C shifted from the original line of sight by an angle $\theta = 40^0$ (θ is known as 'parallax') estimate the distance of the tower C from his original position A.

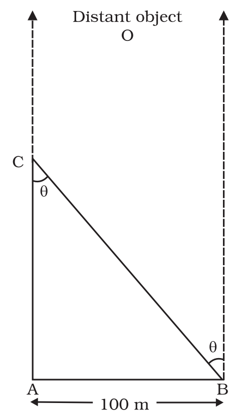

Fig. 2.3

Answer We have, parallax angle $\theta = 40^0$

From Fig. 2.3, AB = AC tan θ

AC = AB/tanθ = 100 m/tan 40^0

= 100 m/0.8391 = 119 m

Example 2.3 The moon is observed from two diametrically opposite points A and B on Earth. The angle θ subtended at the moon by the two directions of observation is $1^\circ 54'$. Given the diameter of the Earth to be about 1.276×10^7 m, compute the distance of the moon from the Earth.

Answer We have $\theta = 1^\circ 54' = 114'$

$$= (114\times60)'' \times (4.85\times10^{-6}) \text{ rad}$$

$$= 3.32\times10^{-2} \text{ rad},$$

since $1'' = 4.85\times10^{-6}$ *rad.*

Also $b = \text{AB} = 1.276\times10^7$ m

Hence from Eq. (2.1), we have the earth-moon distance,

$$D = b/\theta$$

$$= \frac{1.276\times10^7}{3.32\times10^{-2}}$$

$$= 3.84\times10^8 \text{ m}$$

Example 2.4 The Sun's angular diameter is measured to be 1920″. The distance *D* of the Sun from the Earth is 1.496×10^{11} m. What is the diameter of the Sun ?

Answer Sun's angular diameter α

$$= 1920''$$

$$= 1920 \times 4.85 \times 10^{-6} \text{ rad}$$

$$= 9.31 \times 10^{-3} \text{ rad}$$

Sun's diameter

$$d = \alpha D$$

$$= \left(9.31 \times 10^{-3}\right) \times \left(1.496 \times 10^{11}\right) \text{ m}$$

$$= 1.39 \times 10^{9} \text{ m}$$ ◀

2.3.2 Estimation of Very Small Distances: Size of a Molecule

To measure a very small size, like that of a molecule (10^{-8} m to 10^{-10} m), we have to adopt special methods. We cannot use a screw gauge or similar instruments. Even a microscope has certain limitations. An optical microscope uses visible light to 'look' at the system under investigation. As light has wave like features, the resolution to which an optical microscope can be used is the wavelength of light (A detailed explanation can be found in the Class XII Physics textbook). For visible light the range of wavelengths is from about 4000 Å to 7000 Å (1 angstrom = 1 Å = 10^{-10} m). Hence an optical microscope cannot resolve particles with sizes smaller than this. Instead of visible light, we can use an electron beam. Electron beams can be focussed by properly designed electric and magnetic fields. The resolution of such an electron microscope is limited finally by the fact that electrons can also behave as waves ! (You will learn more about this in class XII). The wavelength of an electron can be as small as a fraction of an angstrom. Such electron microscopes with a resolution of 0.6 Å have been built. They can almost resolve atoms and molecules in a material. In recent times, tunnelling microscopy has been developed in which again the limit of resolution is better than an angstrom. It is possible to estimate the sizes of molecules.

A simple method for estimating the molecular size of oleic acid is given below. Oleic acid is a soapy liquid with large molecular size of the order of 10^{-9} m.

The idea is to first form mono-molecular layer of oleic acid on water surface.

We dissolve 1 cm³ of oleic acid in alcohol to make a solution of 20 cm³. Then we take 1 cm³ of this solution and dilute it to 20 cm³, using alcohol. So, the concentration of the solution is equal to $\left(\frac{1}{20 \times 20}\right)$ cm³ of oleic acid/cm³ of solution. Next we lightly sprinkle some lycopodium powder on the surface of water in a large trough and we put one drop of this solution in the water. The oleic acid drop spreads into a thin, large and roughly circular film of molecular thickness on water surface. Then, we quickly measure the diameter of the thin film to get its area A. Suppose we have dropped n drops in the water. Initially, we determine the approximate volume of each drop (V cm³).

Volume of n drops of solution

$$= nV \text{ cm}^3$$

Amount of oleic acid in this solution

$$= nV\left(\frac{1}{20 \times 20}\right) \text{cm}^3$$

This solution of oleic acid spreads very fast on the surface of water and forms a very thin layer of thickness t. If this spreads to form a film of area A cm², then the thickness of the film

$$t = \frac{\text{Volume of the film}}{\text{Area of the film}}$$

or,
$$t = \frac{nV}{20 \times 20\, A} \text{cm} \qquad (2.3)$$

If we assume that the film has mono-molecular thickness, then this becomes the size or diameter of a molecule of oleic acid. The value of this thickness comes out to be of the order of 10^{-9} m.

▶ ***Example 2.5*** If the size of a nucleus (in the range of 10^{-15} to 10^{-14} m) is scaled up to the tip of a sharp pin, what roughly is the size of an atom ? Assume tip of the pin to be in the range 10^{-5}m to 10^{-4}m.

Answer The size of a nucleus is in the range of 10^{-15} m and 10^{-14} m. The tip of a sharp pin is taken to be in the range of 10^{-5} m and 10^{-4} m. Thus we are scaling up by a factor of 10^{10}. An atom roughly of size 10^{-10} m will be scaled up to a size of 1 m. Thus a nucleus in an atom is as small in size as the tip of a sharp pin placed at the centre of a sphere of radius about a metre long. ◀

2.3.3 Range of Lengths

The sizes of the objects we come across in the universe vary over a very wide range. These may vary from the size of the order of 10^{-14} m of the tiny nucleus of an atom to the size of the order of 10^{26} m of the extent of the observable universe. Table 2.3 gives the range and order of lengths and sizes of some of these objects.

We also use certain special length units for short and large lengths. These are

1 fermi = 1 f = 10^{-15} m

1 angstrom = 1 Å = 10^{-10} m

1 astronomical unit = 1 AU (average distance of the Sun from the Earth) = 1.496×10^{11} m

1 light year = 1 ly= 9.46×10^{15} m (distance that light travels with velocity of 3×10^{8} m s^{-1} in 1 year)

1 parsec = 3.08×10^{16} m (Parsec is the distance at which average radius of earth's orbit subtends an angle of 1 arc second)

2.4 MEASUREMENT OF MASS

Mass is a basic property of matter. It does not depend on the temperature, pressure or location of the object in space. The SI unit of mass is kilogram (kg). It is defined by taking the fixed numerical value of the Plank Constant h to be 6.62607015×10^{-34} when expressed in the unit of Js which is equal to kg m^2s^{-1}, where the metre and the second are defined is terms of C and $\Delta\nu$cs.

While dealing with atoms and molecules, the kilogram is an inconvenient unit. In this case, there is an important standard unit of mass, called the **unified atomic mass unit** (u), which has been established for expressing the mass of atoms as

1 unified atomic mass unit = 1u
= (1/12) of the mass of an atom of carbon-12 isotope $\left({}^{12}_{6}C\right)$ including the mass of electrons
= 1.66×10^{-27} kg

Mass of commonly available objects can be determined by a common balance like the one used in a grocery shop. Large masses in the universe like planets, stars, etc., based on Newton's law of gravitation can be measured by using gravitational method (See Chapter 8). For measurement of small masses of atomic/sub-atomic particles etc., we make use of mass spectrograph in which radius of the trajectory is proportional to the mass of a charged particle moving in uniform electric and magnetic field.

2.4.1 Range of Masses

The masses of the objects, we come across in the universe, vary over a very wide range. These may vary from tiny mass of the order of 10^{-30} kg of an electron to the huge mass of about 10^{55} kg of the known universe. Table 2.4 gives the range and order of the typical masses of various objects.

Table 2.3 Range and order of lengths

Size of object or distance	Length (m)
Size of a proton	10^{-15}
Size of atomic nucleus	10^{-14}
Size of hydrogen atom	10^{-10}
Length of typical virus	10^{-8}
Wavelength of light	10^{-7}
Size of red blood corpuscle	10^{-5}
Thickness of a paper	10^{-4}
Height of the Mount Everest above sea level	10^{4}
Radius of the Earth	10^{7}
Distance of moon from the Earth	10^{8}
Distance of the Sun from the Earth	10^{11}
Distance of Pluto from the Sun	10^{13}
Size of our galaxy	10^{21}
Distance to Andromeda galaxy	10^{22}
Distance to the boundary of observable universe	10^{26}

Table 2.4 Range and order of masses

Object	Mass (kg)
Electron	10^{-30}
Proton	10^{-27}
Uranium atom	10^{-25}
Red blood cell	10^{-13}
Dust particle	10^{-9}
Rain drop	10^{-6}
Mosquito	10^{-5}
Grape	10^{-3}
Human	10^{2}
Automobile	10^{3}
Boeing 747 aircraft	10^{8}
Moon	10^{23}
Earth	10^{25}
Sun	10^{30}
Milky way galaxy	10^{41}
Observable Universe	10^{55}

2.5 MEASUREMENT OF TIME

To measure any time interval we need a clock. We now use an **atomic standard of time**, which is based on the periodic vibrations produced in a cesium atom. This is the basis of the **caesium clock**, sometimes called **atomic clock**, used in the national standards. Such standards are available in many laboratories. In the caesium atomic clock, the second is taken as the time needed for 9,192,631,770 vibrations of the radiation corresponding to the transition between the two hyperfine levels of the ground state of caesium-133 atom. The vibrations of the caesium atom regulate the rate of this caesium atomic clock just as the vibrations of a balance wheel regulate an ordinary wristwatch or the vibrations of a small quartz crystal regulate a quartz wristwatch.

The caesium atomic clocks are very accurate. In principle they provide portable standard. The national standard of time interval 'second' as well as the frequency is maintained through four cesium atomic clocks. A caesium atomic clock is used at the National Physical Laboratory (NPL), New Delhi to maintain the Indian standard of time.

In our country, the NPL has the responsibility of maintenance and improvement of physical standards, including that of time, frequency, etc. Note that the Indian Standard Time (IST) is linked to this set of atomic clocks. The efficient caesium atomic clocks are so accurate that they impart the uncertainty in time realisation as $\pm 1 \times 10^{-15}$, i.e. 1 part in 10^{15}. This implies that the uncertainty gained over time by such a device is less than 1 part in 10^{15}; they lose or gain no more than 32 µs in one year. In view of the tremendous accuracy in time measurement, the SI unit of length has been expressed in terms the path length light travels in certain interval of time (1/299, 792, 458 of a second) (Table 2.1).

The time interval of events that we come across in the universe vary over a very wide range. Table 2.5 gives the range and order of some typical time intervals.

You may notice that there is an interesting coincidence between the numbers appearing in Tables 2.3 and 2.5. Note that the ratio of the longest and shortest lengths of objects in our universe is about 10^{41}. Interestingly enough, the ratio of the longest and shortest time intervals associated with the events and objects in our universe is also about 10^{41}. This number, 10^{41} comes up again in Table 2.4, which lists typical masses of objects. The ratio of the largest and smallest masses of the objects in our universe is about $(10^{41})^2$. Is this a curious coincidence between these large numbers purely accidental ?

2.6 ACCURACY, PRECISION OF INSTRUMENTS AND ERRORS IN MEASUREMENT

Measurement is the foundation of all experimental science and technology. The result of every measurement by any measuring instrument contains some uncertainty. This uncertainty is called **error**. Every calculated quantity which is based on measured values, also has an error. We shall distinguish between two terms: **accuracy** and **precision**. The accuracy of a measurement is a measure of how close the measured value is to the true value of the quantity. Precision tells us to what resolution or limit the quantity is measured.

The accuracy in measurement may depend on several factors, including the limit or the resolution of the measuring instrument. For example, suppose the true value of a certain length is near 3.678 cm. In one experiment, using a measuring instrument of resolution 0.1 cm, the measured value is found to be 3.5 cm, while in another experiment using a measuring device of greater resolution, say 0.01 cm, the length is determined to be 3.38 cm. The first measurement has more accuracy (because it is