Chapter Five

MAGNETISM AND MATTER

5.1 Introduction

Magnetic phenomena are universal in nature. Vast, distant galaxies, the tiny invisible atoms, humans and beasts all are permeated through and through with a host of magnetic fields from a variety of sources. The earth's magnetism predates human evolution. The word magnet is derived from the name of an island in Greece called *magnesia* where magnetic ore deposits were found, as early as 600 BC. Shepherds on this island complained that their wooden shoes (which had nails) at times stayed struck to the ground. Their iron-tipped rods were similarly affected. This attractive property of magnets made it difficult for them to move around.

The directional property of magnets was also known since ancient times. A thin long piece of a magnet, when suspended freely, pointed in the north-south direction. A similar effect was observed when it was placed on a piece of cork which was then allowed to float in still water. The name *lodestone* (or *loadstone*) given to a naturally occurring ore of iron-magnetite means leading stone. The technological exploitation of this property is generally credited to the Chinese. Chinese texts dating 400 BC mention the use of magnetic needles for navigation on ships. Caravans crossing the Gobi desert also employed magnetic needles.

A Chinese legend narrates the tale of the victory of the emperor Huang-ti about four thousand years ago, which he owed to his craftsmen (whom

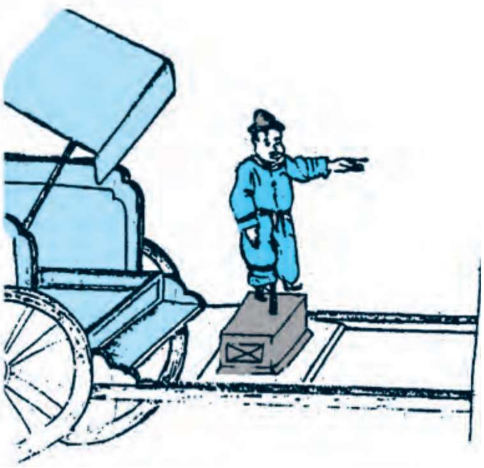

FIGURE 5.1 The arm of the statuette mounted on the chariot always points south. This is an artist's sketch of one of the earliest known compasses, thousands of years old.

nowadays you would call engineers). These 'engineers' built a chariot on which they placed a magnetic figure with arms outstretched. Figure 5.1 is an artist's description of this chariot. The figure swiveled around so that the finger of the statuette on it always pointed south. With this chariot, Huang-ti's troops were able to attack the enemy from the rear in thick fog, and to defeat them.

In the previous chapter we have learned that moving charges or electric currents produce magnetic fields. This discovery, which was made in the early part of the nineteenth century is credited to Oersted, Ampere, Biot and Savart, among others.

In the present chapter, we take a look at magnetism as a subject in its own right.

Some of the commonly known ideas regarding magnetism are:

(i) The earth behaves as a magnet with the magnetic field pointing approximately from the geographic south to the north.

(ii) When a bar magnet is freely suspended, it points in the north-south direction. The tip which points to the geographic north is called the *north pole* and the tip which points to the geographic south is called the *south pole* of the magnet.

(iii) There is a repulsive force when north poles (or south poles) of two magnets are brought close together. Conversely, there is an attractive force between the north pole of one magnet and the south pole of the other.

(iv) We cannot isolate the north, or south pole of a magnet. If a bar magnet is broken into two halves, we get two similar bar magnets with somewhat weaker properties. Unlike electric charges, isolated magnetic north and south poles known as *magnetic monopoles* do not exist.

(v) It is possible to make magnets out of iron and its alloys.

We begin with a description of a bar magnet and its behaviour in an external magnetic field. We describe Gauss's law of magnetism. We then follow it up with an account of the earth's magnetic field. We next describe how materials can be classified on the basis of their magnetic properties. We describe para-, dia-, and ferromagnetism. We conclude with a section on electromagnets and permanent magnets.

5.2 The Bar Magnet

One of the earliest childhood memories of the famous physicist Albert Einstein was that of a magnet gifted to him by a relative. Einstein was fascinated, and played endlessly with it. He wondered how the magnet could affect objects such as nails or pins placed away from it and not in any way *connected* to it by a spring or string.

We begin our study by examining iron filings sprinkled on a sheet of glass placed over a short bar magnet. The arrangement of iron filings is shown in Fig. 5.2.

The pattern of iron filings suggests that the magnet has two poles similar to the positive and negative charge of an electric dipole. As mentioned in the introductory section, one pole is designated the *North pole* and the other, the *South pole*. When suspended freely, these poles point approximately towards the geographic north and south poles, respectively. A similar pattern of iron filings is observed around a current carrying solenoid.

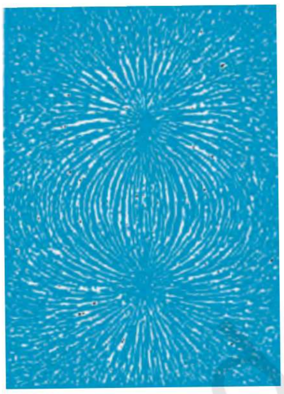

FIGURE 5.2 The arrangement of iron filings surrounding a bar magnet. The pattern mimics magnetic field lines. The pattern suggests that the bar magnet is a magnetic dipole.

5.2.1 The magnetic field lines

The pattern of iron filings permits us to plot the magnetic field lines*. This is shown both for the bar-magnet and the current-carrying solenoid in Fig. 5.3. For comparison refer to the Chapter 1, Figure 1.17(d). Electric field lines of an electric dipole are also displayed in Fig. 5.3(c). The magnetic field lines are a visual and intuitive realisation of the magnetic field. Their properties are:

(i) The magnetic field lines of a magnet (or a solenoid) form continuous closed loops. This is unlike the electric dipole where these field lines begin from a positive charge and end on the negative charge or escape to infinity.

(ii) The tangent to the field line at a given point represents the direction of the net magnetic field **B** at that point.

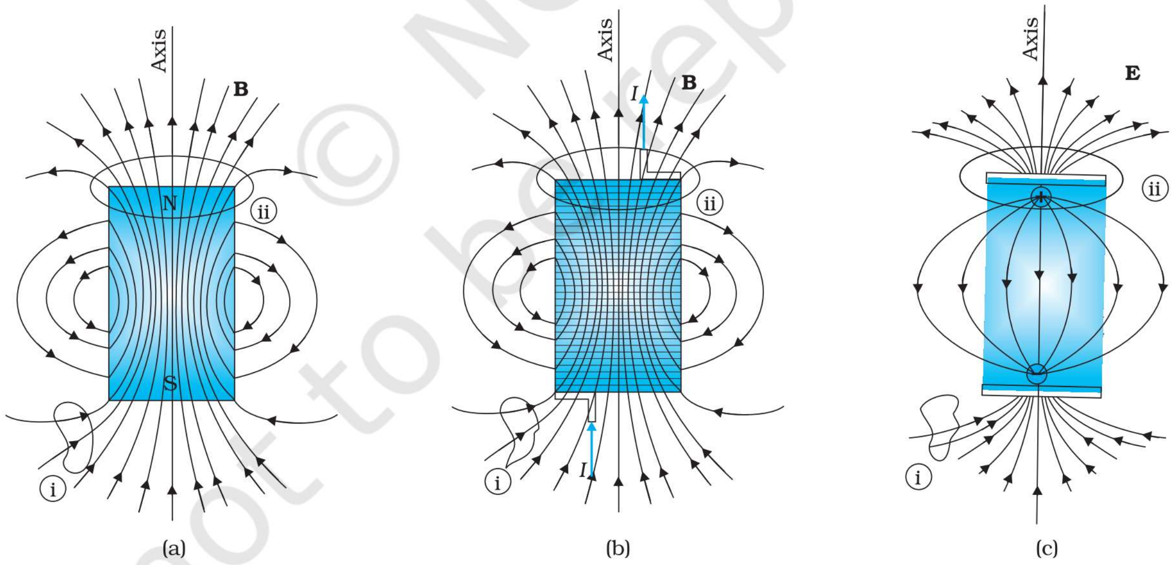

FIGURE 5.3 The field lines of (a) a bar magnet, (b) a current-carrying finite solenoid and (c) electric dipole. At large distances, the field lines are very similar. The curves labelled (i) and (ii) are closed Gaussian surfaces.

* In some textbooks the magnetic field lines are called *magnetic lines of force*. This nomenclature is avoided since it can be confusing. Unlike electrostatics the field lines in magnetism do not indicate the direction of the force on a (moving) charge.

(iii) The larger the number of field lines crossing per unit area, the stronger is the magnitude of the magnetic field **B**. In Fig. 5.3(a), **B** is larger around region (ii) than in region (i).

(iv) The magnetic field lines do not intersect, for if they did, the direction of the magnetic field would not be unique at the point of intersection.

One can plot the magnetic field lines in a variety of ways. One way is to place a small magnetic compass needle at various positions and note its orientation. This gives us an idea of the magnetic field direction at various points in space.

5.2.2 Bar magnet as an equivalent solenoid

In the previous chapter, we have explained how a current loop acts as a magnetic dipole (Section 4.10). We mentioned Ampere's hypothesis that all magnetic phenomena can be explained in terms of circulating currents.

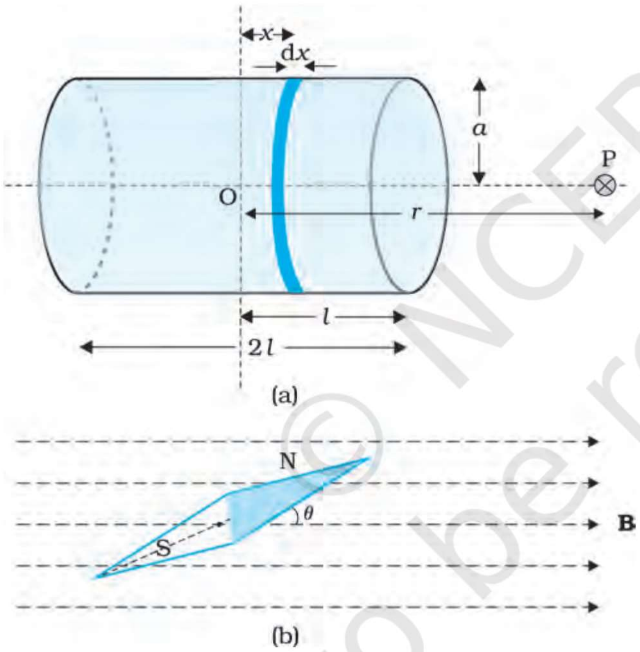

FIGURE 5.4 Calculation of (a) The axial field of a finite solenoid in order to demonstrate its similarity to that of a bar magnet. (b) A magnetic needle in a uniform magnetic field **B**. The arrangement may be used to determine either **B** or the magnetic moment **m** of the needle.

Recall that the magnetic dipole moment **m** associated with a current loop was defined to be **m** = NI**A** where N is the number of turns in the loop, I the current and **A** the area vector (Eq. 4.30).

The resemblance of magnetic field lines for a bar magnet and a solenoid suggest that a bar magnet may be thought of as a large number of circulating currents in analogy with a solenoid. Cutting a bar magnet in half is like cutting a solenoid. We get two smaller solenoids with weaker magnetic properties. The field lines remain continuous, emerging from one face of the solenoid and entering into the other face. One can test this analogy by moving a small compass needle in the neighbourhood of a bar magnet and a current-carrying finite solenoid and noting that the deflections of the needle are similar in both cases.

To make this analogy more firm we calculate the axial field of a finite solenoid depicted in Fig. 5.4 (a). We shall demonstrate that at large distances this axial field resembles that of a bar magnet.

Let the solenoid of Fig. 5.4(a) consists of n turns per unit length. Let its length be $2l$ and radius a. We can evaluate the axial field at a point P, at a distance r from the centre O of the solenoid. To do this, consider a circular element of thickness dx of the solenoid at a distance x from its centre. It consists of $n\,dx$ turns. Let I be the current in the solenoid. In Section 4.6 of the previous chapter we have calculated the magnetic field on the axis of a circular current loop. From Eq. (4.13), the magnitude of the field at point P due to the circular element is

$$dB = \frac{\mu_0 n dx I a^2}{2[(r-x)^2 + a^2]^{3/2}}$$

The magnitude of the total field is obtained by summing over all the elements — in other words by integrating from $x = -l$ to $x = +l$. Thus,

$$B = \frac{\mu_0 n I a^2}{2} \int_{-l}^{l} \frac{dx}{[(r-x)^2 + a^2]^{3/2}}$$

This integration can be done by trigonometric substitutions. This exercise, however, is not necessary for our purpose. Note that the range of x is from $-l$ to $+l$. Consider the far axial field of the solenoid, i.e., $r >> a$ and $r >> l$. Then the denominator is approximated by

$$[(r-x)^2 + a^2]^{3/2} \approx r^3$$

and $B = \dfrac{\mu_0 n I a^2}{2r^3} \displaystyle\int_{-l}^{l} dx$

$$= \frac{\mu_0 n I}{2} \frac{2 l a^2}{r^3} \qquad (5.1)$$

Note that the magnitude of the magnetic moment of the solenoid is, $m = n\,(2\,l)\,I\,(\pi a^2)$ — (total number of turns × current × cross-sectional area). Thus,

$$B = \frac{\mu_0}{4\pi} \frac{2m}{r^3} \qquad (5.2)$$

This is also the far axial magnetic field of a bar magnet which one may obtain experimentally. Thus, a bar magnet and a solenoid produce similar magnetic fields. The magnetic moment of a bar magnet is thus equal to the magnetic moment of an equivalent solenoid that produces the same magnetic field.

Some textbooks assign a *magnetic charge* (also called *pole strength*) $+q_m$ to the north pole and $-q_m$ to the south pole of a bar magnet of length $2l$, and magnetic moment $q_m(2l)$. The field strength due to q_m at a distance r from it is given by $\mu_0 q_m/4\pi r^2$. The magnetic field due to the bar magnet is then obtained, both for the axial and the equatorial case, in a manner analogous to that of an electric dipole (Chapter 1). The method is simple and appealing. However, *magnetic monopoles do not exist*, and *we have avoided this approach for that reason.*

5.2.3 The dipole in a uniform magnetic field

The pattern of iron filings, i.e., the magnetic field lines gives us an approximate idea of the magnetic field **B**. We may at times be required to determine the magnitude of **B** accurately. This is done by placing a small compass needle of known magnetic moment **m** and moment of inertia $\mathcal{I}$ and allowing it to oscillate in the magnetic field. This arrangement is shown in Fig. 5.4(b).

The torque on the needle is [see Eq. (4.29)],

$$\boldsymbol{\tau} = \mathbf{m} \times \mathbf{B} \qquad (5.3)$$

In magnitude $\tau = mB\sin\theta$

Here $\boldsymbol{\tau}$ is restoring torque and θ is the angle between **m** and **B**.

Therefore, in equilibrium $\mathscr{I}\dfrac{d^2\theta}{dt^2} = -mB\sin\theta$

Negative sign with $mB\sin\theta$ implies that restoring torque is in opposition to deflecting torque. For small values of θ in radians, we approximate $\sin\theta \approx \theta$ and get

$$\mathscr{I}\frac{d^2\theta}{dt^2} \approx -mB\,\theta$$

or, $$\frac{d^2\theta}{dt^2} = -\frac{mB}{\mathscr{I}}\theta$$

This represents a simple harmonic motion. The square of the angular frequency is $\omega^2 = mB/\mathscr{I}$ and the time period is,

$$T = 2\pi\sqrt{\frac{\mathscr{I}}{mB}} \qquad (5.4)$$

or $$B = \frac{4\pi^2\,\mathscr{I}}{m\,T^2} \qquad (5.5)$$

An expression for magnetic potential energy can also be obtained on lines similar to electrostatic potential energy.

The magnetic potential energy U_m is given by

$$U_m = \int \tau(\theta)d\theta$$

$$= \int mB\sin\theta\,d\theta = -mB\cos\theta$$

$$= -\mathbf{m}.\mathbf{B} \qquad (5.6)$$

We have emphasised in Chapter 2 that the zero of potential energy can be fixed at one's convenience. Taking the constant of integration to be zero means fixing the zero of potential energy at $\theta = 90°$, i.e., when the needle is perpendicular to the field. Equation (5.6) shows that potential energy is minimum (= –mB) at $\theta = 0°$ (most stable position) and maximum (= +mB) at $\theta = 180°$ (most unstable position).

Example 5.1 In Fig. 5.4(b), the magnetic needle has magnetic moment 6.7×10^{-2} Am^2 and moment of inertia $\mathscr{I} = 7.5 \times 10^{-6}$ kg m^2. It performs 10 complete oscillations in 6.70 s. What is the magnitude of the magnetic field?

Solution The time period of oscillation is,

$$T = \frac{6.70}{10} = 0.67s$$

From Eq. (5.5)

$$B = \frac{4\pi^2\mathscr{I}}{mT^2}$$

$$= \frac{4\times(3.14)^2\times 7.5\times10^{-6}}{6.7\times10^{-2}\times(0.67)^2}$$

$$= 0.01\text{ T}$$

EXAMPLE 5.1