Evaluate $\int \frac{x^2-1}{(x^2+1)\sqrt{1+x^4}}dx$

$$\begin{aligned} I &= \int \frac{x^2-1}{(x^2+1)\sqrt{x^4+1}}dx \\ &= \int \frac{x^2(1-1/x^2)}{x^2(x+1/x)\sqrt{x^2+1/x^2}}dx \\ &= \int \frac{(1-1/x^2)dx}{(x+1/x)\sqrt{(x+1/x)^2-2}} \end{aligned}$$

Putting $x + 1/x = t$, we have

$I = \int \frac{dt}{t\sqrt{t^2-2}}$.
Again putting $t^2 - 2 = y^2, 2tdt = 2ydy$,

$I = \int \frac{ydy}{(y^2+2)y} = \frac{1}{\sqrt{2}}\tan^{-1}\frac{y}{\sqrt{2}} = \frac{1}{2}\tan^{-1}\frac{\sqrt{x^2+1/x^2}}{\sqrt{2}} + c.$