

Evaluate $\int \frac{1}{(1-x^2)\sqrt{1+x^2}} dx$

Putting $x = \frac{1}{t}$ and $dx = -\frac{1}{t^2} dt$, we get

$$\begin{aligned} I &= \int \frac{\left(-\frac{1}{t^2}\right)dt}{\left(1-\frac{1}{t^2}\right)\sqrt{1+\frac{1}{t^2}}} \\ &= -\int \frac{tdt}{(t^2-1)\sqrt{t^2+1}} \end{aligned}$$

Let $t^2 + 1 = u^2$, or $2tdt = 2udu$

$$\begin{aligned} I &= -\int \frac{du}{u^2 - (\sqrt{2})^2} \\ &= -\frac{1}{2\sqrt{2}} \log \left| \frac{u - \sqrt{2}}{u + \sqrt{2}} \right| + C \\ &= -\frac{1}{2\sqrt{2}} \log \left| \frac{\sqrt{t^2 + 1} - \sqrt{2}}{\sqrt{t^2 + 1} + \sqrt{2}} \right| + C \\ &= -\frac{1}{2\sqrt{2}} \log \left| \frac{\sqrt{\frac{1}{x^2} + 1} - \sqrt{2}}{\sqrt{\frac{1}{x^2} + 1} + \sqrt{2}} \right| + C \\ &= -\frac{1}{2\sqrt{2}} \log \left| \frac{\sqrt{1 + x^2} - \sqrt{2}x}{\sqrt{1 + x^2} + \sqrt{2}x} \right| + C \end{aligned}$$