

Evaluate  $\int \frac{1}{(x+1)\sqrt{x^2-1}} dx$

Let  $I = \int \frac{1}{(x+1)\sqrt{x^2-1}} dx$

Putting  $x + 1 = \frac{1}{t}$  then  $dx = -\frac{1}{t^2} dt$ , we get

$$\begin{aligned} I &= \int \frac{1}{\frac{1}{t}\sqrt{\left(\frac{1}{t}-1\right)^2-1}} \left(-\frac{1}{t^2}\right) dt \\ &= -\int \frac{dt}{\sqrt{1-2t}} = -\int (1-2t)^{-1/2} dt \\ &= -\frac{(1-2t)^{1/2}}{(-2)\left(\frac{1}{2}\right)} = \sqrt{1-2t} \\ &= \sqrt{1-\frac{2}{x+1}} + C = \sqrt{\frac{x-1}{x+1}} + C \end{aligned}$$