

Solve: $\int \frac{e^x}{e^{2x} + 6e^x + 5} dx$.

$$I = \int \frac{e^x}{e^{2x} + 6e^x + 5} dx = \int \frac{e^x}{(e^x)^2 + 6e^x + 5} dx$$

Let $e^x = t$ or $e^x dx = dt$

$$\begin{aligned} \therefore I &= \int \frac{dt}{t^2 + 6t + 5} \\ &= \int \frac{1}{(t+3)^2 - 2^2} dt \\ &= \frac{1}{2 \times 2} \log \left| \frac{t+3-2}{t+3+2} \right| = \frac{1}{4} \log \left| \frac{e^x + 1}{e^x + 5} \right| + C \end{aligned}$$