

Solving a System of Linear Equations using Rank of Matrix.

Let 'A' be a $n \times n$ matrix, 'x' be a $n \times 1$ vector, and 'b' be a $n \times 1$ vector. Then the system of n linear equations in variable $x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ can be written as $Ax = b$ ——— ①

Any x which satisfies ① is said to be solution of system of the linear equations.

A system of linear equations can have

i) Unique solution ii) Infinitely many solution iii) No solution

The number of non-zero rows of matrix in Row-Echelon form gives the rank of a matrix.

$\text{Rank}(A)$ can be obtained by reducing matrix A , using elementary row operations, to Row echelon form.

i. If $\text{rank}(A) = \text{rank}(A|b) = n$

Then system of equations has a unique solution.

In this case $\det A \neq 0$.

ii. If $\text{rank}(A) = \text{rank}(A|b)$

$$= m < n.$$

Then the system of equations have infinitely many solutions.

iii. If $\text{rank}(A) \neq \text{rank}(A|b)$

Then the system of equations have no solutions.

$$a^3 + b^3 + c^3 = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca) + 3abc.$$

$$a^2 + b^2 + c^2 = ab + bc + ca$$

$$\Rightarrow 2 \times (a^2 + b^2 + c^2) = 2ab + 2bc + 2ca$$

$$\Rightarrow a^2 - 2ab + b^2 + b^2 - 2bc + c^2 + c^2 - 2ca + a^2 = 0$$

$$\Rightarrow (a-b)^2 + (b-c)^2 + (c-a)^2 = 0$$

$$\Rightarrow a = b = c$$