**Homogeneous Differential Equations**

# Homogeneous Differential Equation

A homogeneous differential equation is an equation containing a differentiation and a function, with a set of variables. The function f(x, y) in a homogeneous differential equation is a homogeneous function such that f(λx, λy) = λnf(x, y), for any non zero constant λ. The general form of a homogeneous differential equation is f(x, y).dy + g(x, y).dx = 0.

Let us learn more about the homogeneous differential equation, the method to solve a homogeneous differential equation, examples .

A first order [Differential Equation](https://www.mathsisfun.com/calculus/differential-equations.html) is **Homogeneous** when it can be in this form:

*dy/***dx** = F(*y/***x** )

We can solve it using [Separation of Variables](https://www.mathsisfun.com/calculus/separation-variables.html) but first we create a new variable **v = *y/*x**

v = *y/***x**   *which is also*   y = vx

And *dy/***dx** = *d (vx)/***dx** = v(*dx/***dx)** + x(*dv/***dx)** (by the [Product Rule](https://www.mathsisfun.com/calculus/derivatives-rules.html))

Which can be simplified to *dy/***dx** = v + x(*dv/***dx)**

Using **y = vx** and ***dy/*dx = v + x(*dv/*dx)**  we can solve the Differential Equation.

## What Is A Homogeneous Differential Equation?

A differential equation containing a homogeneous function is called a homogeneous differential equation. The function f(x, y) is called a homogeneous function if f(λx, λy) = λnf(x, y), for any non zero constant λ. The general form of the homogeneous differential equation is of the form f(x, y).dy + g(x, y).dx = 0. The homogeneous differential equation has the same degree for the variables x, y within the equation.

The homogeneous differential equation does not have a constant term within the equation. The linear differential equation has a constant term. The solution of a linear differential equation is possible if we are able to remove the constant term from the linear differential equation and transform it into a homogeneous differential equation. Also, the homogeneous differential equation does not have the variables x, y within any special functions such as logarithmic, or trigonometric functions.

### Examples of Homogeneous Differential equations.

* dy/dx = (x + y)/(x - y)
* dy/dx = x(x - y)/y2
* dy/dx = (x2 + y2)/xy
* dy/dx = (3x + y)/(x - y)
* dy/dx = (x3 + y3)/(xy2 + yx2)

In these above examples, we can substitute x = λx, and y = λy, to prove it for the homogeneous differential equation. Also, if the homogeneous differential equation is of the form dx/dy = f(x, y), and f(x, y) is a homogeneous function, then we substitute x/y = v, or x = vy. This on further integration, and substituting back the variables x, y, gives the general solution of the homogeneous differential equation.

## How To Solve a Homogeneous Differential Equation?

The general solution of the homogeneous differential equation can be obtained by the integration of the given differential equation. A homogeneous differential equation of the form dy/dx = f(x, y), is solved by first separating the variable and the derivative of the particular variable on either side and then integrating it with respect to the variable.

To solve a homogeneous differential equation of the form dy/dx = f(x, y), we make the substitution y = v.x. Here it is easy to integrate and solve with this substitution. Further the differentiation of y = vx, with respect to x we get dy/dx = v + x.dv/dx. We can substitute the value of dy/dx in the expression dy/dx = f(x, y) = g(y/x) to get the below expression.

v + x. dv/dx = g(v)

x . dv/dx = g(v) - v

Separating the variables x and v, we have:

dv/(g(v)−v)=dx/x

Here we integrate it on both sides, which results in the following expression.

∫1/(g(v)−v).dv=∫1/x.dx

The above expression gives the following solution, which is the general solution of the differential equation.

∫1/(g(v)−v).dv=Logx+C

Here we substitute back the value of v = y/x, to obtain the general solution of the homogeneous differential equation. The presence of +C in the solution, refers it as a general solution, and further solving and substituting the value of +C, we can obtain the particular solution of the given homogeneous differential equation.