A series LCR circuit driven by 300 V at a frequency of 50 Hz contains a resistance R = 3 kΩ, an inductor of inductive reactance X_L = 250 $\pi\Omega$ and an unknown capacitor. The value of capacitance to maximize the average power should be : (Take π^2 = 10) (JEE MAIN 2021)

(A) 4 μF

(B) 25 μF

(C) 400 μF

(D) 40 μF

1. In given LCR circuit;

$V_m = 300V$, $\nu = 50Hz$, $R = 3k\Omega$, $X_L = 250\pi\ \Omega$

$\therefore \omega = 2\pi\nu \Rightarrow 100\pi$ rad/s

Let 'c' be the capacitance of required capacitor.

$\therefore \boxed{X_C = \frac{1}{\omega C} = \frac{1}{(100\pi)C}\ \Omega}$; $\boxed{X_L = 250\pi\ \Omega}$

$$\therefore Z = \sqrt{R^2 + (X_C - X_L)^2}$$
$$= \sqrt{(3\times10^3)^2 + \left[\frac{1}{(100\pi C)} - (250\pi)\right]^2}$$

$$\therefore P_{av} = (\mathcal{E}_{rms})(I_{rms})\cos\phi$$
$$= \left(\frac{\mathcal{E}_0}{\sqrt{2}}\right)\left(\frac{I_0}{\sqrt{2}}\right)\frac{R}{Z} \qquad \left(\because \cos\phi = \frac{R}{Z}\right)$$
$$= \left(\frac{\mathcal{E}_0}{\sqrt{2}}\right)\left(\frac{\mathcal{E}_0}{\sqrt{2}\,Z}\right)\frac{R}{Z}$$
$$P_{av} = \frac{\mathcal{E}_0^2 R}{2Z^2}$$

(☆ Trick :- No need to evaluate whole expression. Just take derivative w.r.t. C and evaluate it.)

As $P_{av} \propto \frac{1}{Z^2}$

$\Rightarrow$ For P_{av} to be max, Z^2 must be minimum

$\Rightarrow$ $Z^2 = (3\times10^3)^2 + \left(\dfrac{1}{100\pi C} - 250\pi\right)^2$ is minimum

which is possible only when $X_C - X_L = 0$

i.e. $\dfrac{1}{100\pi C} - 250\pi = 0$

$\Rightarrow C = \dfrac{1}{(100\pi)(250\pi)}$

$\Rightarrow C = \dfrac{1}{25\times10^3\times10}$ (Taking $\pi^2 = 10$)

$\Rightarrow \boxed{C = 4\,\mu F}$