## 1. Circle

A circle is the locus of a point whose distance is constant from a given point around which it moves.

**Equation of circle**

A circle with center (a, b) and radius r is given as

$$\mathbf{(x + a)^2 + (y + b)^2 = r^2}$$

## 3. The general equation of the circle

Any second order curve can be defined as

$Ax^2+Bxy+Cy^2+Gx+Hy+D=0$

The equation represents a circle on the conditions:

A=C and B=0.

Thus, the general equation of the circle is

$Ax^2+Ay^2+Dx+Ey+G=0$

Or

$$\mathbf{x^2+y^2+2gx+2hy+c=0}$$

Rewrite the equation of the circle

$x^2+y^2+2gx+2hy+c=0$

$x^2+y^2+2gx+2hy+c+g^2+h^2 = g^2+h^2$

$(x + g)^2+(y + h)^2= g^2+h^2-c$

The center of the circle is (-g,-h) and the radius of the circle is $\sqrt{g^2+h^2-c}$

**Note: For a real circle** $g^2+h^2-c\geq0$

## 4.Diametric form of a circle

If the endpoints of a diameter to the circle are $(x_1,y_1)$ and $(x_2,y_2)$

$$\left(\frac{y-y_1}{x-x_1}\right)\left(\frac{y-y_2}{x-x_2}\right)=-1$$

**Position of a point w.r.t circle**

If S: $x^2 + y^2 + 2gx + 2hy + c = 0$

Let point be P ($x_1$, $y_1$)

$S_p$: $x_1^2 + y_1^2 + 2gx_1 + 2hy_1 + c = 0$

If $S_p > 0$ point lies outside the circle.

If $S_p = 0$ point lies on the circle.

If $S_p < 0$ point lies inside the circle.

If the point lies outside the circle, the greatest distance of the point from the circle is

(CP + r) and the least distance of the point from the circle is (CP – r).

**Parametric equation of the circle**

The parametric coordinates of the circle are ($x_1 + r\cos\theta$ , $y_1 + r\sin\theta$ ) where the ($x_1$ , $y_1$) is the centre of the circle and r is the radius of the circle, is the angle made by the point from the centre.

The parametric equation is given by

$$\frac{x-x_1}{\cos\theta}=\frac{y-y_1}{\sin\theta}=r$$

**Line and Circle**

Solve the equation of circle by substituting any one variable from the equation of the line. A quadratic equation in any on a variable is formed. Evaluate the determinant of the equation.

If the discriminant, $D > 0$, the line is a secant.

If the discriminant, $D=0$, the line is a tangent.

If the discriminant, $D<0$, then the line doesn't touch the circle.

**Equation of tangent of the circle**

**At a point $(x_1, y_1)$**

Form the equation of tangent by replacing

$x^2 \rightarrow xx_1$

$y^2 \rightarrow yy_1$

$x \rightarrow (x+x_1)/2$

$y \rightarrow (y+y_1)/2$

in the equation of the circle.

**Slope form**

If the equation of the tangent is y=mx + c, where m is the slope of the tangent and c is arbitrary constant, then Slope of the tangent to a circle $x^2 + y^2 = r^2$ is

$$y = mx \pm a\sqrt{1+m^2}$$

**9.Length of the tangent**

If the equation of circle is given as, **S: $x^2 + y^2 + 2gx + 2hy + c = 0$,** the length of tangent to the point $P(x_1,y_1)$ is given by

$L_T = \sqrt{S_P}$

Where $S_p$: $x_1^2 + y_1^2 + 2gx_1 + 2hy_1 + c$

**10. The equation of chord of a circle when the median of the chord is given.**

If the median of the chord is (h, k), then the equation of chord to the circle

S: $(x + a)^2 + (y + b)^2 = r^2$

is

$xh+yk-r^2=h^2+k^2$

**or**

**$(T=S_P)$**

**11. Equation of chord of contact**

The equation of chord from the contact to the circle S: $(x + a)^2 + (y + b)^2 = r^2$ is

$xh+yk-r^2=0$

**12. Equation of pair of tangents**

If the circle is given as S: $x^2 + y^2 = r^2$

$S_p$: $x_1^2 + y_1^2 = r^2$

$T_P$: $xx_1+yy_1 = r^2$

$SS_P=T_P^2$

**13. Family of circles passing the points of intersection of two circles**

If the circle S' and S'' are intersecting, the family of circles that can be formed from the point of contacts S' and S'' is

$S' + \lambda S''=0$

**14. Family of circles passing the points of intersection of a circle and a line**

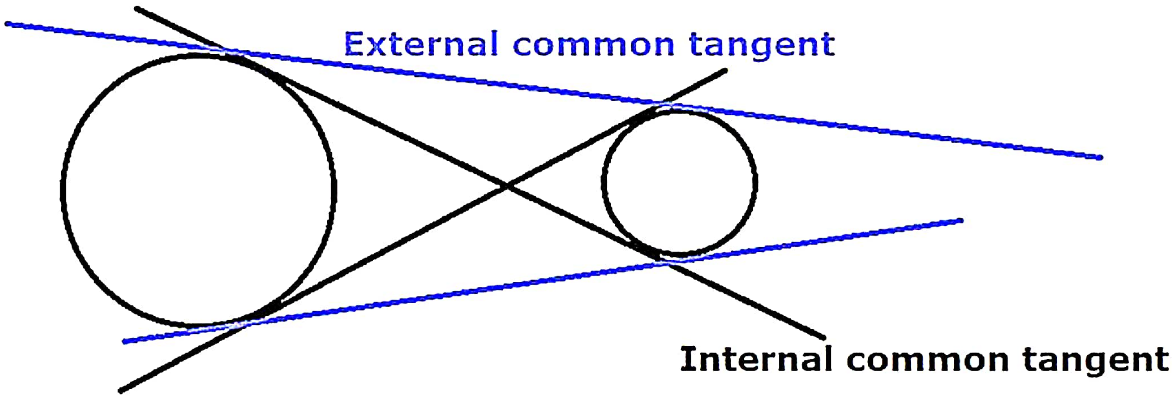
If the circle S and line L are intersecting, the family of circles that can be formed from the point of contacts S and L is

**Common tangent to two circles**

There are two types of common tangents to two circles,

Internal common tangent

External common tangent

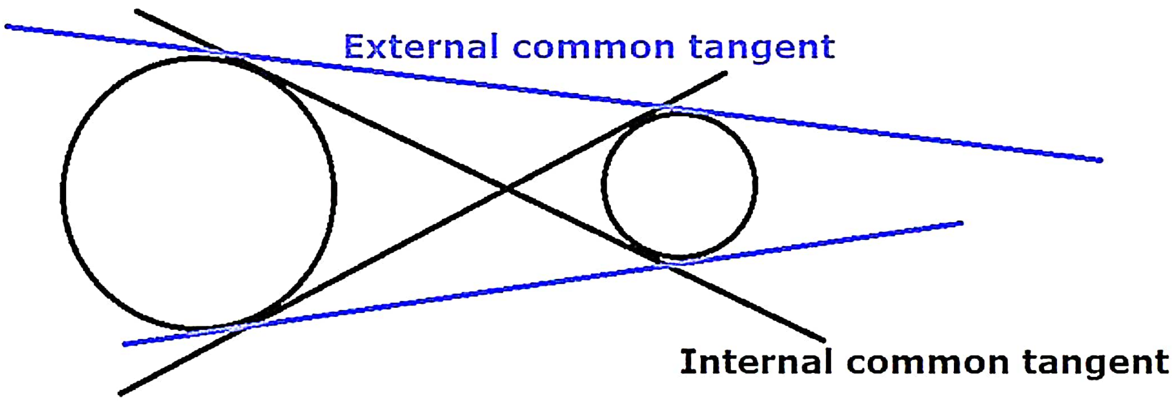



**16. Position of circles and the number of common tangents**

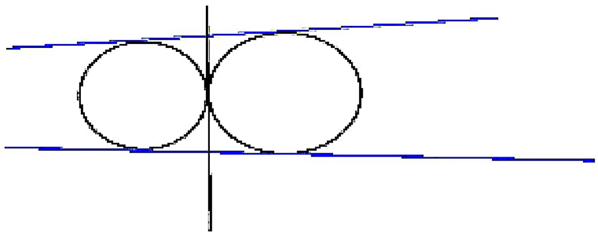
Let $C_1$ and $C_2$ denote the centre of the circles, $r_1$ & $r_2$ be the radii of the circles and $C_1C_2$ be the distance between the centres.

(a) If $C_1C_2 > r_1 + r_2$

There are 4 common tangents

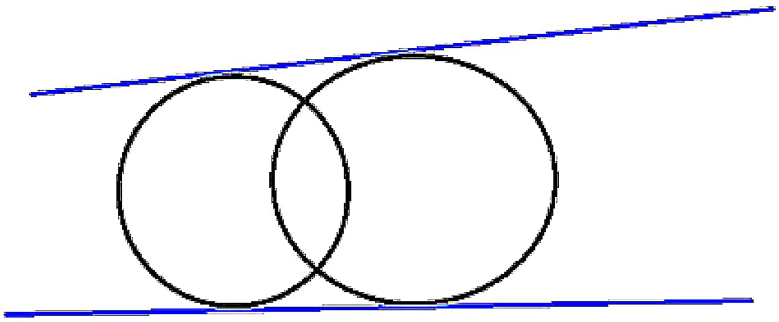



(b) If $C_1C_2 = r_1 + r_2$



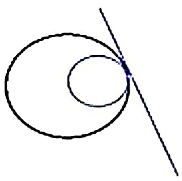
There are 3 common tangents.

(c) If $|r_1 - r_2| < C_1C_2 < r_1 + r_2$



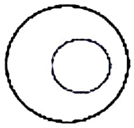
There are 2 common tangents.

(d) If $C_1C_2 = |r_1 - r_2|$



There is one common tangent.

(e) If $C_1C_2 < |r_1 - r_2|$



There is no common tangent.

**17. Length of common tangent**

**(a) External tangent**

$$L_{ET} = \sqrt{(C_1C_2)^2 - (r_1 - r_2)^2}$$

**(b) Internal tangent**

$$L_{IT} = \sqrt{(C_1C_2)^2 - (r_1 + r_2)^2}$$

**18. The condition of orthogonality of two circles**

Two circles intersect each other at 90° when

$2g_1g_2 + 2f_1f_2 = c_1+c_2$