A fully charged capacitor C with initial charge q_0 is connected to a coil of self inductance L at $t = 0$. The time at which the energy is stored equally between the electric and the magnetic fields is : (JEE MAIN 2011)

A. $\frac{\pi}{4}\sqrt{LC}$

B. $2\pi\sqrt{LC}$

C. $\sqrt{LC}$

D. $\pi\sqrt{LC}$

4. Initial charge on capacitor = q_0

Let time varying charge in given AC - circuit be

$q = q_0 \sin(\omega t)$

Also, $i = \frac{dq}{dt} \Rightarrow \frac{d}{dt}[q_0 \sin(\omega t)]$

$\Rightarrow \quad i = (q_0\omega)\cos(\omega t)$

$\Rightarrow \quad i = i_0 \cos\omega t \qquad$ (where $i_0 = q_0\omega$)

Given, magnetic energy = electrostatic energy

$\Rightarrow \quad \frac{1}{2} L i^2 = \frac{1}{2}\frac{q^2}{C}$

$\Rightarrow \quad L[i_0^2 \cos^2(\omega t)] = q_0^2 \frac{\sin^2(\omega t)}{C}$

$\Rightarrow (LC)\, \cancel{q_0^2}\, \omega^2 \cos^2(\omega t) = \cancel{q_0^2} \sin^2 \omega t$

(Here, $\omega = \frac{1}{\sqrt{LC}}$ because $X_C = X_L$ at resonant frequency)

$\Rightarrow \quad \tan^2(\omega t) = 1$

$\Rightarrow \quad \omega t = \frac{\pi}{4}$

$\Rightarrow \quad \boxed{t = \frac{\pi}{4}\sqrt{LC}}$