Example 10.1 The two thigh bones (femurs), each of cross-sectional area 10 cm^2 support the upper part of a human body of mass 40 kg. Estimate the average pressure sustained by the femurs.

Answer Total cross-sectional area of the femurs is $A = 2 \times 10\ cm^2 = 20 \times 10^{-4}\ m^2$. The force acting on them is F = 40 kg wt = 400 N (taking g = 10 m s^{-2}). This force is acting vertically down and hence, normally on the femurs. Thus, the average pressure is

$$P_{av} = \frac{F}{A} = 2 \times 10^5\ N\ m^{-2}$$

10.2.1 Pascal's Law

The French scientist Blaise Pascal observed that the pressure in a fluid at rest is the same at all points if they are at the same height. This fact may be demonstrated in a simple way.

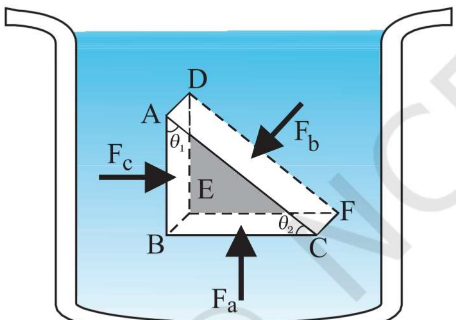


Fig. 10.2 *Proof of Pascal's law. ABC-DEF is an element of the interior of a fluid at rest. This element is in the form of a right-angled prism. The element is small so that the effect of gravity can be ignored, but it has been enlarged for the sake of clarity.*

Fig. 10.2 shows an element in the interior of a fluid at rest. This element ABC-DEF is in the form of a right-angled prism. In principle, this prismatic element is very small so that every part of it can be considered at the same depth from the liquid surface and therefore, the effect of the gravity is the same at all these points. But for clarity we have enlarged this element. The forces on this element are those exerted by the rest of the fluid and they must be normal to the surfaces of the element as discussed above. Thus, the fluid exerts pressures P_a, P_b and P_c on this element of area corresponding to the normal forces F_a, F_b and F_c as shown in Fig. 10.2 on the faces BEFC, ADFC and ADEB denoted by A_a, A_b and A_c respectively. Then

$F_b \sin\theta = F_c$, $F_b \cos\theta = F_a$ (by equilibrium)
$A_b \sin\theta = A_c$, $A_b \cos\theta = A_a$ (by geometry)

Thus,

$$\frac{F_b}{A_b} = \frac{F_c}{A_c} = \frac{F_a}{A_a}; \quad P_b = P_c = P_a \tag{10.4}$$

Hence, pressure exerted is same in all directions in a fluid at rest. It again reminds us that like other types of stress, pressure is not a vector quantity. No direction can be assigned to it. The force against any area within (or bounding) a fluid at rest and under pressure is normal to the area, regardless of the orientation of the area.

Now consider a fluid element in the form of a horizontal bar of uniform cross-section. The bar is in equilibrium. The horizontal forces exerted at its two ends must be balanced or the pressure at the two ends should be equal. This proves that for a liquid in equilibrium the pressure is same at all points in a horizontal plane. Suppose the pressure were not equal in different parts of the fluid, then there would be a flow as the fluid will have some net force acting on it. Hence in the absence of flow the pressure in the fluid must be same everywhere in a horizontal plane.

10.2.2 Variation of Pressure with Depth

Consider a fluid at rest in a container. In Fig. 10.3 point 1 is at height h above a point 2. The pressures at points 1 and 2 are P_1 and P_2 respectively. Consider a cylindrical element of fluid having area of base A and height h. As the fluid is at rest the resultant horizontal forces should be zero and the resultant vertical forces should balance the weight of the element. The forces acting in the vertical direction are due to the fluid pressure at the top (P_1A) acting downward, at the bottom (P_2A) acting upward. If mg is weight of the fluid in the cylinder we have

$$(P_2 - P_1)\ A = mg \tag{10.5}$$

Now, if ρ is the mass density of the fluid, we have the mass of fluid to be $m = \rho V = \rho h A$ so that

$$P_2 - P_1 = \rho g h \tag{10.6}$$

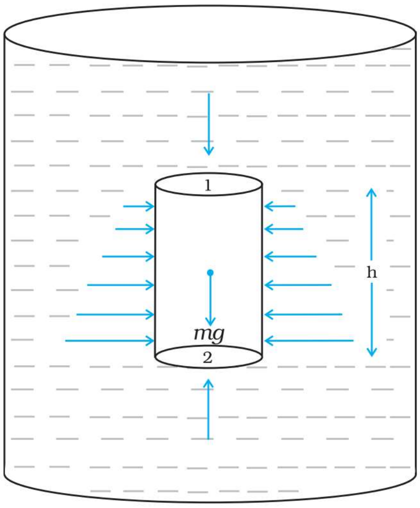


Fig.10.3 *Fluid under gravity. The effect of gravity is illustrated through pressure on a vertical cylindrical column.*

Pressure difference depends on the vertical distance h between the points (1 and 2), mass density of the fluid ρ and acceleration due to gravity g. If the point 1 under discussion is shifted to the top of the fluid (say, water), which is open to the atmosphere, P_1 may be replaced by atmospheric pressure (P_a) and we replace P_2 by P. Then Eq. (10.6) gives

$$P = P_a + \rho gh \tag{10.7}$$

Thus, the pressure P, at depth below the surface of a liquid open to the atmosphere is greater than atmospheric pressure by an amount ρgh. The excess of pressure, $P - P_a$, at depth h is called a **gauge pressure** at that point.

The area of the cylinder is not appearing in the expression of absolute pressure in Eq. (10.7). Thus, the height of the fluid column is important and not cross-sectional or base area or the shape of the container. The liquid pressure is the same at all points at the same horizontal level (same depth). The result is appreciated through the example of **hydrostatic paradox**. Consider three vessels A, B and C [Fig.10.4] of different shapes. They are connected at the bottom by a horizontal pipe. On filling with water, the level in the three vessels is the same, though they hold different amounts of water. This is so because water at the bottom has the same pressure below each section of the vessel.

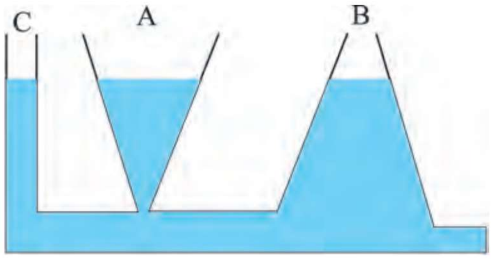


Fig 10.4 *Illustration of hydrostatic paradox. The three vessels A, B and C contain different amounts of liquids, all upto the same height.*

Example 10.2 What is the pressure on a swimmer 10 m below the surface of a lake?

Answer Here

h = 10 m *and* ρ = 1000 kg m^{-3}. Take g = 10 m s^{-2}

From Eq. (10.7)

$P = P_a + \rho gh$

$= 1.01 \times 10^5$ Pa + 1000 kg m^{-3} × 10 m s^{-2} × 10 m

$= 2.01 \times 10^5$ Pa

$\approx$ 2 atm

This is a 100% increase in pressure from surface level. At a depth of 1 km, the increase in pressure is 100 atm! Submarines are designed to withstand such enormous pressures. ◀

10.2.3 Atmospheric Pressure and Gauge Pressure

The pressure of the atmosphere at any point is equal to the weight of a column of air of unit cross-sectional area extending from that point to the top of the atmosphere. At sea level, it is 1.013×10^5 Pa (1 atm). Italian scientist Evangelista Torricelli (1608–1647) devised for the first time a method for measuring atmospheric pressure. A long glass tube closed at one end and filled with mercury is inverted into a trough of mercury as shown in Fig.10.5 (a). This device is known as 'mercury barometer'. The space above the mercury column in the tube contains only mercury vapour whose pressure P is so small that it may be neglected. Thus, the pressure at Point A=0. The pressure inside the coloumn at Point B must be the same as the pressure at Point C, which is atmospheric pressure, P_a.

$$P_a = \rho gh \tag{10.8}$$

where ρ is the density of mercury and h is the height of the mercury column in the tube.

In the experiment it is found that the mercury column in the barometer has a height of about 76 cm at sea level equivalent to one atmosphere (1 atm). This can also be obtained using the value of ρ in Eq. (10.8). A common way of stating pressure is in terms of cm or mm of mercury (Hg). A pressure equivalent of 1 mm is called a torr (after Torricelli).

1 torr = 133 Pa.

The mm of Hg and torr are used in medicine and physiology. In meteorology, a common unit is the bar and millibar.

1 bar = 10^5 Pa

An open tube manometer is a useful instrument for measuring pressure differences. It consists of a U-tube containing a suitable liquid i.e., a low density liquid (such as oil) for measuring small pressure differences and a high density liquid (such as mercury) for large pressure differences. One end of the tube is open to the atmosphere and the other end is connected to the system whose pressure we want to measure [see Fig. 10.5 (b)]. The pressure P at A is equal to pressure at point B. What we normally measure is the gauge pressure, which is $P - P_a$, given by Eq. (10.8) and is proportional to manometer height h.

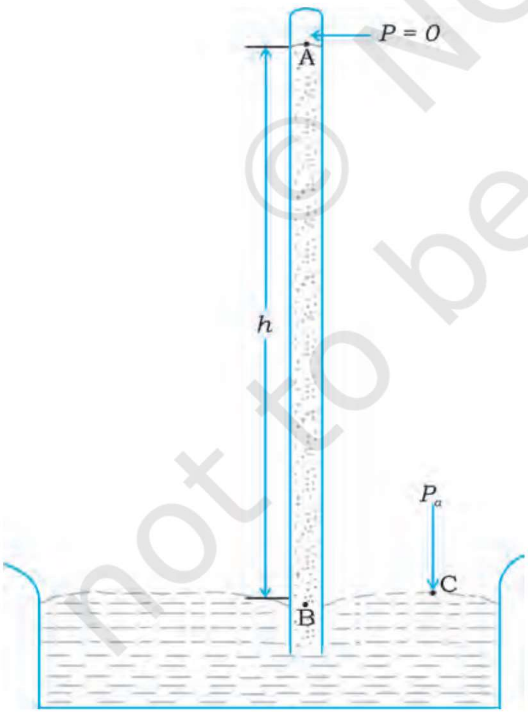
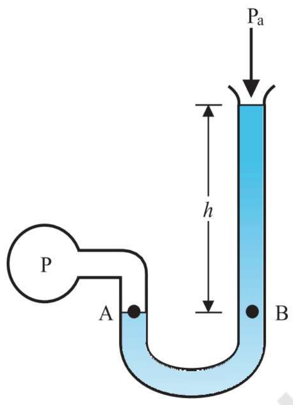


Fig 10.5 *(a) The mercury barometer.*



(b) The open tube manometer

Fig 10.5 *Two pressure measuring devices.*

Pressure is same at the same level on both sides of the U-tube containing a fluid. For liquids, the density varies very little over wide ranges in pressure and temperature and we can treat it safely as a constant for our present purposes. Gases on the other hand, exhibits large variations of densities with changes in pressure and temperature. Unlike gases, liquids are, therefore, largely treated as incompressible.

Example 10.3 The density of the atmosphere at sea level is 1.29 kg/m^3. Assume that it does not change with altitude. Then how high would the atmosphere extend?

Answer We use Eq. (10.7)

$\rho gh = 1.29 \text{ kg m}^{-3} \times 9.8 \text{ m s}^{2} \times h \text{ m} = 1.01 \times 10^5 \text{ Pa}$

$\therefore h = 7989 \text{ m} \approx 8 \text{ km}$

In reality the density of air decreases with height. So does the value of g. The atmospheric cover extends with decreasing pressure over 100 km. We should also note that the sea level atmospheric pressure is not always 760 mm of Hg. A drop in the Hg level by 10 mm or more is a sign of an approaching storm.

Example 10.4 At a depth of 1000 m in an ocean (a) what is the absolute pressure? (b) What is the gauge pressure? (c) Find the force acting on the window of area 20 cm × 20 cm of a submarine at this depth, the interior of which is maintained at sea-level atmospheric pressure. (The density of sea water is 1.03×10^3 kg m^{-3}, $g = 10$ m s^{-2}.)