Two charges q_1 and q_2 are placed at (0, 0, d) and (0, 0, -d) respectively. Find locus of points where the potential a zero.

Sol. Let the potential at any point P(x, y, z) is zero then $-V_1 + V_2 = 0$

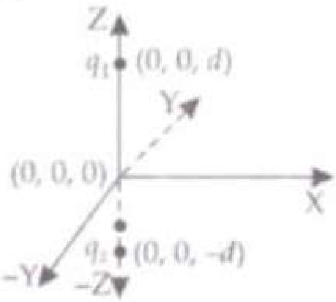
$$\frac{kq_1}{\sqrt{x^2+y^2+(z-d)^2}} + \frac{kq_2}{\sqrt{x^2+y^2+(z+d)^2}} = 0$$

$$\frac{q_1}{\sqrt{x^2+y^2+(z-d)^2}} + \frac{q_2}{\sqrt{x^2+y^2+(z+d)^2}} = 0$$

$$\frac{q_1}{\sqrt{x^2+y^2+(z-d)^2}} + \frac{-q_2}{\sqrt{x^2+y^2+(z+d)^2}}$$

$$\frac{q_1}{q_2} = \frac{-\sqrt{x^2+y^2+(z-d)^2}}{\sqrt{x^2+y^2+(z+d)^2}}$$

$$\frac{q_1^2}{q_2^2} = \frac{x^2+y^2+z^2+d^2-zd}{x^2+y^2+z^2+d^2+zd}$$



Componendo and dividend of $\frac{x}{a} = \frac{y}{b}$ is $\frac{x+a}{x-a} = \frac{y+b}{y-b}$

Then componendo and dividend of

$$\frac{\left(\frac{q_1}{q_2}\right)^2}{1} = \frac{x^2+y^2+z^2+d^2-2zd}{x^2+y^2+z^2+d^2+2dz}$$

$$\frac{\left(\frac{q_1}{q_2}\right)^2+1}{\left(\frac{q_2}{q_2}\right)^2-1} = \frac{x^2+y^2+z^2+d^2-2dz+(x^2+y^2+z^2+d^2+2dz)}{x^2+y^2+z^2+d^2-2dz-(x^2+y^2+z^2+d^2+2dz)}$$

$$x^2 + y^2 + z^2 + d^2 = -2dz\left[\frac{\left(\frac{q_1}{q_2}\right)^2+1}{\left(\frac{q_1}{q_2}\right)^2-1}\right]$$

$$x^2 + y^2 + z^2 = 2d\left[\frac{\left(\frac{q_1}{q_2}\right)^2+1}{\left(\frac{q_1}{q_2}\right)^2-1}\right] z + d^2 = 0$$

$$x^2 + y^2 + z^2 = 2d\left[\frac{q_1^2+q_2^2}{q_1^2-q_2^2}\right] z + d^2 = 0$$

This is the equation of the sphere with centre at

$$\left(0, 0, -2d\left[\frac{q_1^2+q_2^2}{q_1^2-q_2^2}\right]\right)$$

Note: if $q_1 = -q_2$ then $z = 0$, which is a plane through mid-point.

Therefore, locus of points where potential is zero is the plane through mid point of the two charges.