Two charges -q each are separated by distance 2d. A third charge +q is kept at mid-point O. Find potential energy of +q as a function of small distance x from O due to -q charges. Sketch P.E. v/s x and convince yourself that the charge at O is in an unstable equilibrium.

Sol. $V = \frac{kq}{r}$

Let equilibrium of +q is at P at a distance x from mid-point of line joining two charges.

Force F_A on +q is towards left side and force F_B is towards right side, so for equilibrium of +q at P,

$F_A = F_B$

$\frac{-kq^2}{(d-x)^2} = \frac{-kq^2}{(d+x)^2}$

$\therefore (d - x)^2 = (d + x)^2$

d - x = d + x (Taking square root)

-2x = 0

x = 0

So, Equilibrium position of charge +q between two -q charges is at mid-point (O) of line joining the two charges (-q) and (-q).

Now we have to find out potential energy of +q as a function of small distance x from balance condition (O) towards any of (-q) charge.

Let new position of charge (+q) from a small distance x from (O), then we have

$U = \frac{k \cdot (q)(-q)}{(d-x)} + \frac{k(q)(-q)}{(d+x)} \left(\because U = \frac{kq_1q_2}{r_1}\right)$

$= -kq^2\left[\frac{1}{(d-x)} + \frac{1}{(d+x)}\right]$

$= -kq^2\left[\frac{d+x+d-x}{(d-x)(d+x)}\right] = -kq^2\left[\frac{2d}{d^2-x^2}\right]$

$U = \frac{-q^2}{4\pi\varepsilon_0} \cdot \frac{2d}{(d^2-x^2)}$

So, U is the P.E. as a function of x.

x	U
0	$\frac{-2k}{d}q^2$
$\frac{d}{2}$	$\frac{4}{3}\left(\frac{-2kq^2}{d}\right)$
$-\frac{d}{2}$	$\frac{4}{3}\left(\frac{-2kq^2}{d}\right)$
+d	-α
d	-α