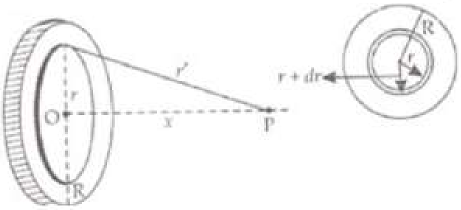
Calculate potential on the axis of a disc of radius R due to a charge Q uniformly distributed on its surface.

Sol. Consider a point P on the axis perpendicular to the plane of disc and at distance x from the center O of the disc as shown in the figure.



Now consider a ring of radius r of thickness dr on a disc of radius R, as shown in the figure, Let us consider disc is divided into a large number of rings, Again let the charge on the ring is dq then potential dV due to ring at P, will be

$dV = \frac{kdq}{r'}\left[\because r' = \sqrt{r^2 + x^2}\right]$

dq is the charge on the ring = σ area of ring

$= \sigma \cdot \left[\pi(r + dr)^2 - \pi r^2\right]$

$dq = \sigma \cdot \pi\left[r^2 + dr^2 + 2rdr - r^2\right]$

Because dr is small, therefore, dr_2 is negligible.

$\therefore \text{dq} = \sigma\pi(2rdr) = 2\pi r\sigma \cdot dr$

$\therefore \text{dV} = \frac{k.2\pi r\sigma dr}{\sqrt{(r^2+x^2)}}$

So the potential due to charged disc

$\int_0^V dV = \int_0^R \frac{k.2\pi\sigma dr}{\sqrt{r^2+x^2}}$

$V = k.2\pi\sigma \cdot \int_0^R \frac{rdr}{(r^2+x^2)^{12}} = k.\,\pi\sigma \int_0^R r \cdot \left(r^2 + x^2\right)^{-1/2} 2dr$ = $\frac{k\pi\sigma[\sqrt{r^2+x^2}]_0^R}{1/2}$

$= 2\pi k\sigma\left[\left(R^2 + x^2\right)^{1/2} - x\right] = \frac{2\pi\sigma}{4\pi\varepsilon_0}\left[\left(R^2 + x^2\right)^{1/2} - x\right]$

[$\because$ $\pi R^2\sigma$ = Q(charge on disc) $\sigma = \frac{Q}{\pi R^2}$

$= \frac{\pi 2R^2\sigma}{4\pi\varepsilon_0 R^2}[\sqrt{R^2 + x^2} - x]$ thus the potential due to a disc is given by,

$V = \frac{2Q}{4\pi\varepsilon_0 R^2}[\sqrt{R^2 + x^2} - x]$