a. In a quark model of elementary particles, a neutron is made of one up quarks [charge $(\frac{2}{3})$ e] and two down quarks [charges $-(\frac{1}{3})$ e]. Assume that they have a triangle configuration with side length of the order of 10^{-15} m. Calculate the electrostatic potential energy of neutron and compare it with its mass 939 MeV.

b. Repeat above exercise for a proton which is made of two up and one down quark.

Sol.

a. $q_d = -\frac{1}{3}e$ [charge on down quark]

$q_u = +\frac{2}{3}e$ [charge on up quark]

Potential energy for charges is given by $U = \frac{kq_1q_2}{r}$

$k = \frac{1}{4\pi\epsilon_0}$

$U = \frac{kq_1q_2}{r} + \frac{kq_1q_3}{r} + \frac{kq_2q_3}{r}$

$\therefore U_n = \frac{1}{4\pi\varepsilon_0}\frac{(-q_d)(-q_d)}{r} + \frac{(-q_d)q_u}{4\pi\varepsilon_0} + \frac{q_u(-q_d)}{4\pi\varepsilon_0 r}$

$= \frac{q_d}{4\pi\varepsilon_0 r}[+q_d - q_u - q_u]$ [Talking sign of charge]

$= \frac{qd}{4\pi\varepsilon_0 r}[q_d - 2q_u] = \frac{9\times10^9\times\frac{1}{3}e}{10^{-15}}\left[\frac{1}{3}e - 2\cdot\frac{2}{3}e\right]$

[nature sign of charges taken already]

$= \frac{9\times10^9\times e}{3\times10^{-15}}\cdot\frac{e}{3}[1-4]$ Joule

$= \frac{-3\times9\times10^9\times1.6\times10^{-15}}{9\times10^{-15}}$ Joule = -7.68×10^{-14} J

$\frac{-7.68X10^{-14}}{1.6X10^{-19}}$ = $-4.8 \times 10^{-14+19}$ e V = 4.8×10^5 e V = -0.48×10^6 e V

U = -0.48 MeV

So, charges inside neutron [1q_u and 2q_d] are attracted by the energy of 0.48 MeV.

The energy released by a neutron when converted into energy is 939 MeV.

$\therefore$ Required ratio $= \frac{1-0.481\text{MeV}}{939\text{MeV}}$ = 0.0005111 = 5.11×10^{-4}

b. P.E. Of proton consists of 2 up and 1 down quark

r = 10^{-15} m

$q_d = -\frac{1}{3}e, q_u = \frac{2}{3}e$

$U_p = \frac{1}{4\pi\varepsilon_0}\frac{q_u\times q_u}{r} + \frac{q_u(-q_d)}{4\pi\varepsilon_0 r} + \frac{q_u(-q_d)}{4\pi\varepsilon_0 r}$

$= \frac{q_u}{4\pi\varepsilon_0 r}[q_u - q_d - q_d]$

$= \frac{q_u}{4\pi\varepsilon_0 r}[q_u - 2q_d] = \frac{9\times10^9}{10^{-15}}\frac{2}{3}e\left[\frac{2}{3}e - 2\cdot\frac{1}{3}e\right] = 0$ potential energy is zero for this case.