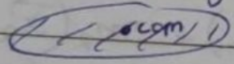
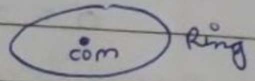
COM

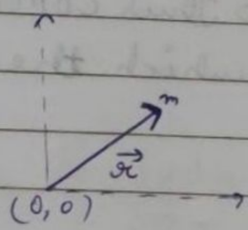
- concept of com is not applicable for rotatory motion.
- com can lie on the body or it may lie outside





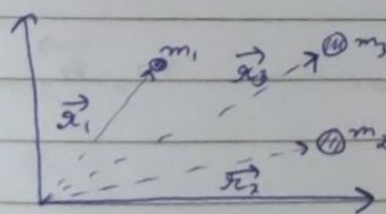
Mass Moment:

Mass moment = $m\vec{r}$



For extended body we take $\vec{r}_{com}$ of object.

Discrete particle system.



$$\vec{r}_{cm} = \frac{m_1\vec{x}_1 + m_2\vec{r}_2 - - - - m_n\vec{x}_n}{m_1 + m_2 - - - - m_n}$$

$$\vec{r}_{cm} = \vec{x}_{cm}\hat{i} + \vec{y}_{cm}\hat{j} + z_{cm}\hat{k}$$

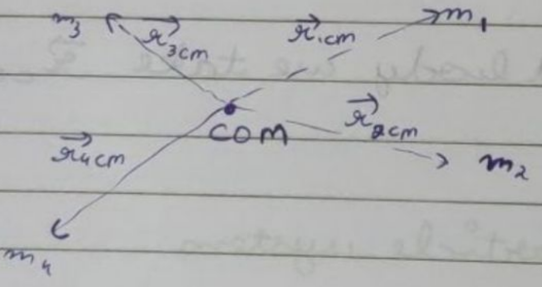
$$x_{cm} = \frac{m_1\vec{x}_1 + m_2 x_2 - - - m_n x_n}{m_1 + m_2 - - - - m_n}$$

$$y_{cm} = \frac{m_1 y_1 + m_2 y_2 - - - m_n y_n}{m_1 + m_2 - - - m_n}$$

$$\vec{r}_{cm} = \frac{m_1 \vec{r_1} + m_2 \vec{r_2} - - - m_n \vec{r_n}}{m_1 + m_2 - - m_n}$$

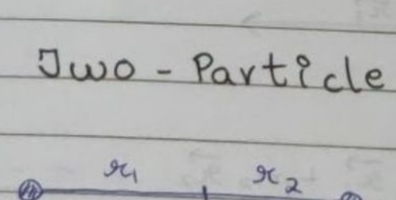
$$0 = m_1(\vec{r_1} - \vec{r}_{cm}) + m_2(\vec{r_2} - \vec{r}_{cm}) - - - m_n(\vec{r_n} - \vec{r}_{cm})$$

$\Rightarrow$ Thus COM is located at a point w.r.t which the mass moment is always zero.



$$m_1 \vec{r}_{1cm} + m_2 \vec{r}_{2cm} - - m_4 \vec{r}_{4cm} = 0$$

Two - Particle System



$$m_1 r_1 = m_2 r_2$$

$$\Rightarrow r_1 = \frac{m_2}{m_1 + m_2} r \quad , \quad r_2 = \frac{m_1 r}{m_1 + m_2}$$

Consider a n-particle system and all particles are shifted by $d\vec{r}_1, d\vec{r}_2$ ———— $d\vec{r}_n$

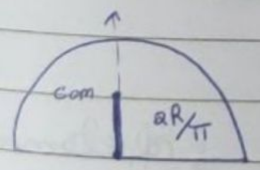
$$d\vec{r}_{cm} = \frac{m_1 d\vec{r}_1 + m_2 d\vec{r}_2 - - - m_n d\vec{r}_n}{m_1 + m_2 - - - m_n}$$

Calculation of COM for a Continuous mass system

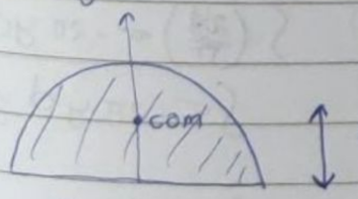
$$x_{cm} = \frac{\int x\,dm}{\int dm} \qquad y_{cm} = \frac{\int y\,dm}{\int dm}$$

$$z_{cm} = \frac{\int z\,dm}{\int dm}$$

Results :

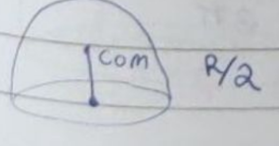
① 

Ring

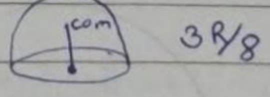
② 

Disc

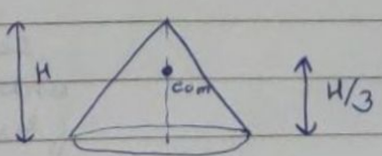
③ Hollow Hemisphere



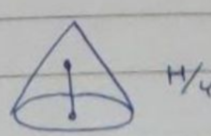
④ Solid Hemisphere



⑤ Hollow Cone



⑥ Solid Cone



Motion of COM

$$\vec{r}_{cm} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 \text{ --- } - m_n \vec{r}_n}{m_1 + m_2 \text{ --- --- } m_n}$$

$$\vec{V}_{cm} = \frac{m_1 \vec{V}_1 + m_2 \vec{V}_2 \text{ --- --- } m_n \vec{V}_n}{m_1 + m_2 \text{ -- } m_n}$$

$$\vec{a}_{cm} = \frac{m_1 \vec{a}_1 + m_2 \vec{a}_2 \text{ - - - } m_n \vec{a}_n}{m_1 + m_2 \text{ - - - } m_n}$$

$$\vec{a}_{cm} = \frac{\vec{F}_{ext}}{m_{Total}}$$ (Internal forces don't affect motion of COM)

- $m_1(\vec{r_1}-\vec{r}_{cm})+m_2(\vec{r_2}-\vec{r}_{cm}) \text{ --- } m_n(\vec{r_n}-\vec{r}_{cm})=0$

$m_1(\vec{V_1}-\vec{V}_{cm})+m_2(\vec{V_2}-\vec{V}_{cm}) \text{ --- } m_n(\vec{V_n}-\vec{V}_{cm})=0$

$m_1(\vec{a_1}-\vec{a}_{cm})+m_2(\vec{a_2}-\vec{a}_{cm}) \text{ --- } m_n(\vec{a_n}-\vec{a}_{cm})=0$

Thus, in COM frame

Net mass moment, Total momentum & Net force are always zero whether Fext is acting ~~of~~ or not.

- Let Fext $=0 \Rightarrow a_{cm}=0 \Rightarrow V_{cm}=$ Const

Hence $m_1\vec{V_1}+m_2\vec{V_2} \text{ ---- } m_n\vec{V_n}=$ Const

This is known as law of conservation of linear momentum.

$\Rightarrow$ Internal forces may change of the momentum of a particle but the net momentum always remains constant.

$\Rightarrow$ momentum conservation is valid both at microscopic and macroscopic level.

momentum conservation for a system of particles

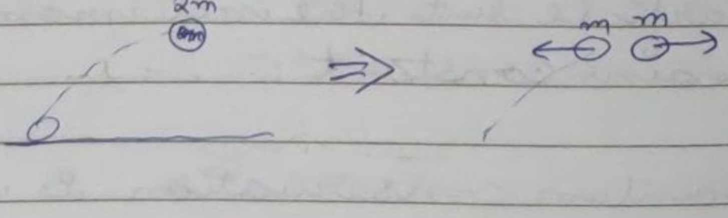
- If velocity of com is const w.r.t time them momentum of system remains const with time.

- In such conservation momentum of individual particle may change but net resultant of momentum is always const.

- This is possible if net $F_{ext} = 0$

Note. If external force is present or non zero then in such case the linear momentum can be conserved only in direction ~~with~~ which is $\perp^{rd}$ to ext force

Eg



momentum conservation can be applied.