every volume element Δv of the slab has a dipole moment $\mathbf{P}\,\Delta v$ in the direction of the field. The volume element Δv is macroscopically small but contains a very large number of molecular dipoles. Anywhere inside the dielectric, the volume element Δv has no net charge (though it has net dipole moment). This is, because, the positive charge of one dipole sits close to the negative charge of the adjacent dipole. However, at the surfaces of the dielectric normal to the electric field, there is evidently a net charge density. As seen in Fig 2.23, the positive ends of the dipoles remain unneutralised at the right surface and the negative ends at the left surface. The unbalanced charges are the induced charges due to the external field.

Thus, the polarised dielectric is equivalent to two charged surfaces with induced surface charge densities, say σ_p and $-\sigma_p$. Clearly, the field produced by these surface charges opposes the external field. The total field in the dielectric is, thereby, reduced from the case when no dielectric is present. We should note that the surface charge density $\pm\sigma_p$ arises from bound (not free charges) in the dielectric.

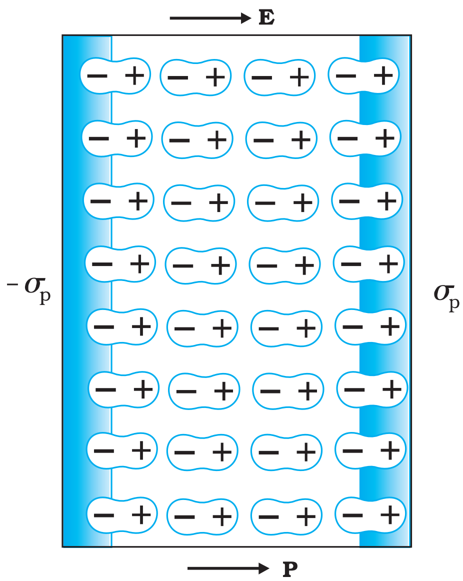

FIGURE 2.23 A uniformly polarised dielectric amounts to induced surface charge density, but no volume charge density.

2.11 Capacitors and Capacitance

A capacitor is a system of two conductors separated by an insulator (Fig. 2.24). The conductors have charges, say Q_1 and Q_2, and potentials V_1 and V_2. Usually, in practice, the two conductors have charges Q and $-Q$, with potential difference $V = V_1 - V_2$ between them. We shall consider only this kind of charge configuration of the capacitor. (Even a single conductor can be used as a capacitor by assuming the other at infinity.) The conductors may be so charged by connecting them to the two terminals of a battery. Q is called the charge of the capacitor, though this, in fact, is the charge on one of the conductors – the total charge of the capacitor is zero.

The electric field in the region between the conductors is proportional to the charge Q. That is, if the charge on the capacitor is, say doubled, the electric field will also be doubled at every point. (This follows from the direct proportionality between field and charge implied by Coulomb's law and the superposition principle.) Now, potential difference V is the work done per unit positive charge in taking a small test charge from the conductor 2 to 1 against the field. Consequently, V is also proportional to Q, and the ratio Q/V is a constant:

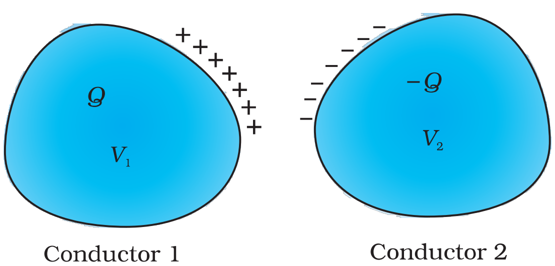

FIGURE 2.24 A system of two conductors separated by an insulator forms a capacitor.

$$C = \frac{Q}{V} \tag{2.38}$$

The constant C is called the *capacitance* of the capacitor. C is independent of Q or V, as stated above. The capacitance C depends only on the geometrical configuration (shape, size, separation) of the system of two conductors. [As we shall see later, it also depends on the nature of the insulator (dielectric) separating the two conductors.] The SI unit of capacitance is 1 farad (=1 coulomb volt^{-1}) or 1 F = 1 C V^{-1}. A capacitor with fixed capacitance is symbolically shown as ⊣ ⊢, while the one with variable capacitance is shown as ⫲.

Equation (2.38) shows that for large *C*, *V* is small for a given *Q*. This means a capacitor with large capacitance can hold large amount of charge *Q* at a relatively small *V*. This is of practical importance. High potential difference implies strong electric field around the conductors. A strong electric field can ionise the surrounding air and accelerate the charges so produced to the oppositely charged plates, thereby neutralising the charge on the capacitor plates, at least partly. In other words, the charge of the capacitor leaks away due to the reduction in insulating power of the intervening medium.

The maximum electric field that a dielectric medium can withstand without break-down (of its insulating property) is called its *dielectric strength*; for air it is about 3×10^6 Vm^{-1}. For a separation between conductors of the order of 1 cm or so, this field corresponds to a potential difference of 3×10^4 V between the conductors. Thus, for a capacitor to store a large amount of charge without leaking, its capacitance should be high enough so that the potential difference and hence the electric field do not exceed the break-down limits. Put differently, there is a limit to the amount of charge that can be stored on a given capacitor without significant leaking. In practice, a farad is a very big unit; the most common units are its sub-multiples 1 μF = 10^{-6} F, 1 nF = 10^{-9} F, 1 pF = 10^{-12} F, etc. Besides its use in storing charge, a capacitor is a key element of most ac circuits with important functions, as described in Chapter 7.

2.12 The Parallel Plate Capacitor

A parallel plate capacitor consists of two large plane parallel conducting plates separated by a small distance (Fig. 2.25). We first take the intervening medium between the plates to be vacuum. The effect of a dielectric medium between the plates is discussed in the next section. Let *A* be the area of each plate and *d* the separation between them. The two plates have charges *Q* and –*Q*. Since *d* is much smaller than the linear dimension of the plates ($d^2 << A$), we can use the result on electric field by an infinite plane sheet of uniform surface charge density (Section 1.15). Plate 1 has surface charge density $\sigma = Q/A$ and plate 2 has a surface charge density $-\sigma$. Using Eq. (1.33), the electric field in different regions is:

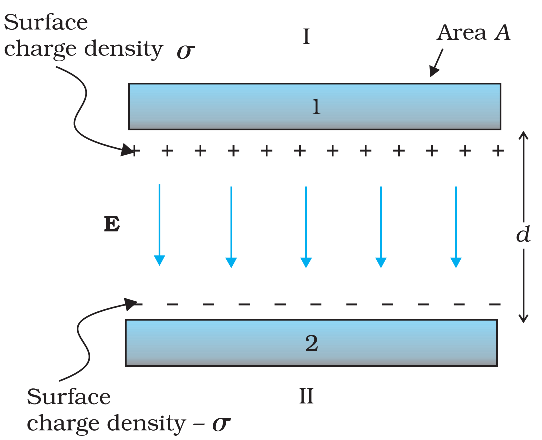

FIGURE 2.25 The parallel plate capacitor.

Outer region I (region above the plate 1),

$$E = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0 \qquad (2.39)$$

Outer region II (region below the plate 2),

$$E = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0 \qquad (2.40)$$

In the inner region between the plates 1 and 2, the electric fields due to the two charged plates add up, giving

$$E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A} \qquad (2.41)$$

The direction of electric field is from the positive to the negative plate.

Thus, the electric field is localised between the two plates and is uniform throughout. For plates with finite area, this will not be true near the outer boundaries of the plates. The field lines bend outward at the edges — an effect called 'fringing of the field'. By the same token, σ will not be strictly uniform on the entire plate. [E and σ are related by Eq. (2.35).] However, for $d^2 << A$, these effects can be ignored in the regions sufficiently far from the edges, and the field there is given by Eq. (2.41). Now for uniform electric field, potential difference is simply the electric field times the distance between the plates, that is,

$$V = E\,d = \frac{1}{\varepsilon_0}\frac{Qd}{A} \qquad (2.42)$$

The capacitance C of the parallel plate capacitor is then

$$C = \frac{Q}{V} = = \frac{\varepsilon_0 A}{d} \qquad (2.43)$$

which, as expected, depends only on the geometry of the system. For typical values like $A = 1\ \text{m}^2$, $d = 1$ mm, we get

$$C = \frac{8.85 \times 10^{-12}\,\text{C}^2\text{N}^{-1}\text{m}^{-2} \times 1\,\text{m}^2}{10^{-3}\,\text{m}} = 8.85 \times 10^{-9}\,\text{F} \qquad (2.44)$$

(You can check that if $1\text{F} = 1\text{C V}^{-1} = 1\text{C}\ (\text{NC}^{-1}\text{m})^{-1} = 1\ \text{C}^2\,\text{N}^{-1}\text{m}^{-1}$.) This shows that 1F is too big a unit in practice, as remarked earlier. Another way of seeing the 'bigness' of 1F is to calculate the area of the plates needed to have C = 1F for a separation of, say 1 cm:

$$A = \frac{Cd}{\varepsilon_0} = \frac{1\text{F} \times 10^{-2}\,\text{m}}{8.85 \times 10^{-12}\,\text{C}^2\text{N}^{-1}\text{m}^{-2}} = 10^9\,\text{m}^2 \qquad (2.45)$$

which is a plate about 30 km in length and breadth!

2.13 Effect of Dielectric on Capacitance

With the understanding of the behaviour of dielectrics in an external field developed in Section 2.10, let us see how the capacitance of a parallel plate capacitor is modified when a dielectric is present. As before, we have two large plates, each of area A, separated by a distance d. The charge on the plates is $\pm Q$, corresponding to the charge density $\pm\sigma$ (with $\sigma = Q/A$). When there is vacuum between the plates,

$$E_0 = \frac{\sigma}{\varepsilon_0}$$

PHYSICS

Factors affecting capacitance, capacitors in action
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http://micro.magnet.fsu.edu/electromag/java/capacitance/

and the potential difference V_0 is

$V_0 = E_0 d$

The capacitance C_0 in this case is

$$C_0 = \frac{Q}{V_0} = \varepsilon_0 \frac{A}{d} \qquad (2.46)$$

Consider next a dielectric inserted between the plates fully occupying the intervening region. The dielectric is polarised by the field and, as explained in Section 2.10, the effect is equivalent to two charged sheets (at the surfaces of the dielectric normal to the field) with surface charge densities σ_p and $-\sigma_p$. The electric field in the dielectric then corresponds to the case when the net surface charge density on the plates is $\pm(\sigma - \sigma_p)$. That is,

$$E = \frac{\sigma - \sigma_P}{\varepsilon_0} \qquad (2.47)$$

so that the potential difference across the plates is

$$V = E\,d = \frac{\sigma - \sigma_P}{\varepsilon_0} d \qquad (2.48)$$

For linear dielectrics, we expect σ_p to be proportional to E_0, i.e., to σ. Thus, $(\sigma - \sigma_p)$ is proportional to σ and we can write

$$\sigma - \sigma_P = \frac{\sigma}{K} \qquad (2.49)$$

where K is a constant characteristic of the dielectric. Clearly, $K > 1$. We then have

$$V = \frac{\sigma d}{\varepsilon_0 K} = \frac{Qd}{A\varepsilon_0 K} \qquad (2.50)$$

The capacitance C, with dielectric between the plates, is then

$$C = \frac{Q}{V} = \frac{\varepsilon_0 KA}{d} \qquad (2.51)$$

The product $\varepsilon_0 K$ is called the *permittivity* of the medium and is denoted by ε

$$\varepsilon = \varepsilon_0 K \qquad (2.52)$$

For vacuum $K = 1$ and $\varepsilon = \varepsilon_0$; ε_0 is called the *permittivity of the vacuum*. The dimensionless ratio

$$K = \frac{\varepsilon}{\varepsilon_0} \qquad (2.53)$$

is called the *dielectric constant* of the substance. As remarked before, from Eq. (2.49), it is clear that K is greater than 1. From Eqs. (2.46) and (2. 51)

$$K = \frac{C}{C_0} \qquad (2.54)$$

Thus, the dielectric constant of a substance is the factor (>1) by which the capacitance increases from its vacuum value, when the dielectric is inserted fully between the plates of a capacitor. Though we arrived at

Eq. (2.54) for the case of a parallel plate capacitor, it holds good for any type of capacitor and can, in fact, be viewed in general as a definition of the dielectric constant of a substance.

ELECTRIC DISPLACEMENT

We have introduced the notion of dielectric constant and arrived at Eq. (2.54), without giving the explicit relation between the induced charge density σ_p and the polarisation **P**.

We take without proof the result that

$$\sigma_P = \mathbf{P} \cdot \hat{\mathbf{n}}$$

where $\hat{\mathbf{n}}$ is a unit vector along the outward normal to the surface. Above equation is general, true for any shape of the dielectric. For the slab in Fig. 2.23, **P** is along $\hat{\mathbf{n}}$ at the right surface and opposite to $\hat{\mathbf{n}}$ at the left surface. Thus at the right surface, induced charge density is positive and at the left surface, it is negative, as guessed already in our qualitative discussion before. Putting the equation for electric field in vector form

$$\mathbf{E} \cdot \hat{\mathbf{n}} = \frac{\sigma - \mathbf{P} \cdot \hat{\mathbf{n}}}{\varepsilon_0}$$

or $(\varepsilon_0 \mathbf{E} + \mathbf{P}) \cdot \hat{\mathbf{n}} = \sigma$

The quantity $\varepsilon_0 \mathbf{E} + \mathbf{P}$ is called the *electric displacement* and is denoted by **D**. It is a vector quantity. Thus,

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}, \ \mathbf{D} \cdot \hat{\mathbf{n}} = \sigma,$$

The significance of **D** is this : in vacuum, **E** is related to the free charge density σ. When a dielectric medium is present, the corresponding role is taken up by **D**. For a dielectric medium, it is **D** not **E** that is directly related to free charge density σ, as seen in above equation. Since **P** is in the same direction as **E**, all the three vectors **P**, **E** and **D** are parallel.

The ratio of the magnitudes of **D** and **E** is

$$\frac{D}{E} = \frac{\sigma\varepsilon_0}{\sigma - \sigma_P} = \varepsilon_0 K$$

Thus,

$$\mathbf{D} = \varepsilon_0 K \mathbf{E}$$

and $\mathbf{P} = \mathbf{D} - \varepsilon_0\mathbf{E} = \varepsilon_0 (K-1)\mathbf{E}$

This gives for the electric susceptibility χ_e defined in Eq. (2.37)

$\chi_e = (K-1)$

EXAMPLE 2.8

Example 2.8 A slab of material of dielectric constant K has the same area as the plates of a parallel-plate capacitor but has a thickness $(3/4)d$, where d is the separation of the plates. How is the capacitance changed when the slab is inserted between the plates?

Solution Let $E_0 = V_0/d$ be the electric field between the plates when there is no dielectric and the potential difference is V_0. If the dielectric is now inserted, the electric field in the dielectric will be $E = E_0/K$. The potential difference will then be

Example 2.8

$$V = E_0(\frac{1}{4}d) + \frac{E_0}{K}(\frac{3}{4}d)$$

$$= E_0 d(\frac{1}{4} + \frac{3}{4K}) = V_0 \frac{K+3}{4K}$$

The potential difference decreases by the factor $(K + 3)/4K$ while the free charge Q_0 on the plates remains unchanged. The capacitance thus increases

$$C = \frac{Q_0}{V} = \frac{4K}{K+3}\frac{Q_0}{V_0} = \frac{4K}{K+3}C_0$$

2.14 Combination of Capacitors

We can combine several capacitors of capacitance C_1, C_2,…, C_n to obtain a system with some effective capacitance C. The effective capacitance depends on the way the individual capacitors are combined. Two simple possibilities are discussed below.

2.14.1 Capacitors in series

Figure 2.26 shows capacitors C_1 and C_2 combined in series.

The left plate of C_1 and the right plate of C_2 are connected to two terminals of a battery and have charges Q and $–Q$, respectively. It then follows that the right plate of C_1 has charge $–Q$ and the left plate of C_2 has charge Q. If this was not so, the net charge on each capacitor would not be zero. This would result in an electric field in the conductor connecting C_1 and C_2. Charge would flow until the net charge on both C_1 and C_2 is zero and there is no electric field in the conductor connecting C_1 and C_2. Thus, in the series combination, charges on the two plates ($\pm Q$) are the same on each capacitor. The total potential drop V across the combination is the sum of the potential drops V_1 and V_2 across C_1 and C_2, respectively.

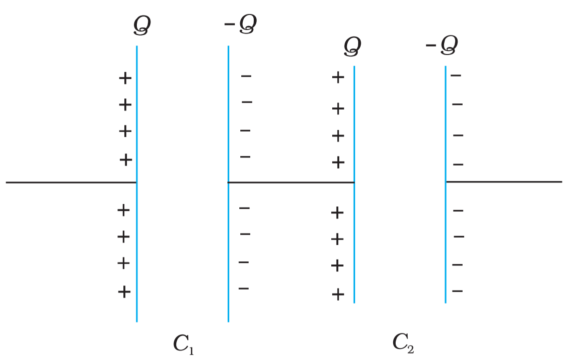

FIGURE 2.26 Combination of two capacitors in series.

$$V = V_1 + V_2 = \frac{Q}{C_1} + \frac{Q}{C_2} \tag{2.55}$$

$$\text{i.e., } \frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2}, \tag{2.56}$$

Now we can regard the combination as an effective capacitor with charge Q and potential difference V. The *effective capacitance* of the combination is

$$C = \frac{Q}{V} \tag{2.57}$$

We compare Eq. (2.57) with Eq. (2.56), and obtain

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \tag{2.58}$$

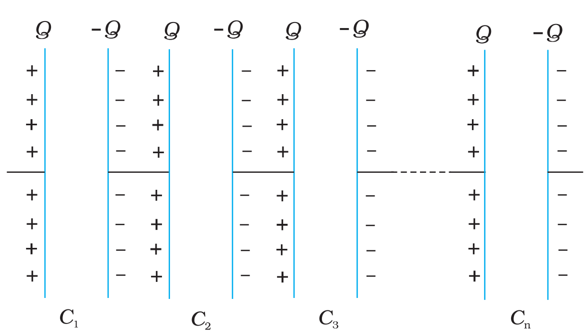

FIGURE 2.27 Combination of n capacitors in series.

The proof clearly goes through for any number of capacitors arranged in a similar way. Equation (2.55), for n capacitors arranged in series, generalises to

$$V = V_1 + V_2 + \ldots + V_n = \frac{Q}{C_1} + \frac{Q}{C_2} + \ldots + \frac{Q}{C_n} \quad (2.59)$$

Following the same steps as for the case of two capacitors, we get the general formula for effective capacitance of a series combination of n capacitors:

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots + \frac{1}{C_n} \quad (2.60)$$

2.14.2 Capacitors in parallel

Figure 2.28 (a) shows two capacitors arranged in parallel. In this case, the same potential difference is applied across both the capacitors. But the plate charges ($\pm Q_1$) on capacitor 1 and the plate charges ($\pm Q_2$) on the capacitor 2 are not necessarily the same:

$$Q_1 = C_1 V, \; Q_2 = C_2 V \quad (2.61)$$

The equivalent capacitor is one with charge

$$Q = Q_1 + Q_2 \quad (2.62)$$

and potential difference V.

$$Q = CV = C_1 V + C_2 V \quad (2.63)$$

The effective capacitance C is, from Eq. (2.63),

$$C = C_1 + C_2 \quad (2.64)$$

The general formula for effective capacitance C for parallel combination of n capacitors [Fig. 2.28 (b)] follows similarly,

$$Q = Q_1 + Q_2 + \ldots + Q_n \quad (2.65)$$

i.e., $CV = C_1 V + C_2 V + \ldots C_n V \quad (2.66)$

which gives

$$C = C_1 + C_2 + \ldots C_n \quad (2.67)$$

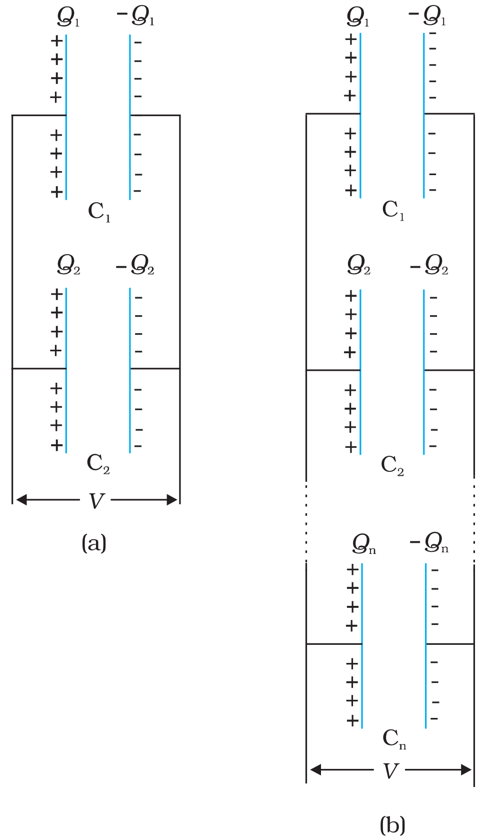

FIGURE 2.28 Parallel combination of (a) two capacitors, (b) n capacitors.

Example 2.9 A network of four 10 μF capacitors is connected to a 500 V supply, as shown in Fig. 2.29. Determine (a) the equivalent capacitance of the network and (b) the charge on each capacitor. (Note, the *charge on a capacitor* is the charge on the plate with higher potential, equal and opposite to the charge on the plate with lower potential.)

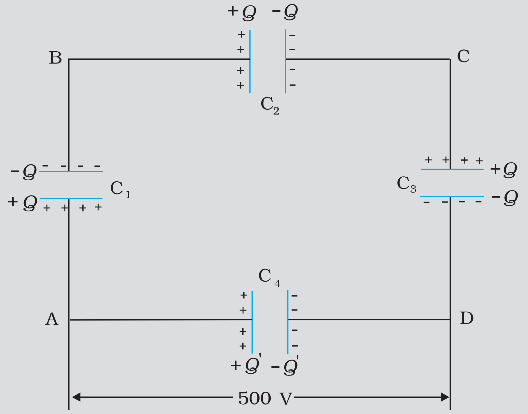

FIGURE 2.29

EXAMPLE 2.9

EXAMPLE 2.9

Solution

(a) In the given network, C_1, C_2 and C_3 are connected in series. The effective capacitance C′ of these three capacitors is given by

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

For $C_1 = C_2 = C_3 = 10$ µF, $C' = (10/3)$ µF. The network has C' and C_4 connected in parallel. Thus, the equivalent capacitance C of the network is

$$C = C' + C_4 = \left(\frac{10}{3} + 10\right) \mu\text{F} = 13.3\mu\text{F}$$

(b) Clearly, from the figure, the charge on each of the capacitors, C_1, C_2 and C_3 is the same, say Q. Let the charge on C_4 be Q'. Now, since the potential difference across AB is Q/C_1, across BC is Q/C_2, across CD is Q/C_3, we have

$$\frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} = 500\,\text{V}$$

Also, $Q'/C_4 = 500$ V.

This gives for the given value of the capacitances,

$$Q = 500\,V \times \frac{10}{3} \mu\text{F} = 1.7 \times 10^{-3}\,\text{C} \text{ and}$$

$$Q' = 500\,V \times 10\,\mu\text{F} = 5.0 \times 10^{-3}\,\text{C}$$

2.15 ENERGY STORED IN A CAPACITOR

A capacitor, as we have seen above, is a system of two conductors with charge Q and $-Q$. To determine the energy stored in this configuration, consider initially two uncharged conductors 1 and 2. Imagine next a process of transferring charge from conductor 2 to conductor 1 bit by bit, so that at the end, conductor 1 gets charge Q. By charge conservation, conductor 2 has charge $-Q$ at the end (Fig 2.30).

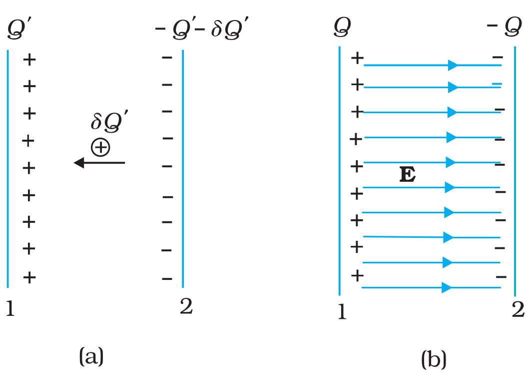

FIGURE 2.30 (a) Work done in a small step of building charge on conductor 1 from Q' to $Q' + \delta Q'$. (b) Total work done in charging the capacitor may be viewed as stored in the energy of electric field between the plates.

In transferring positive charge from conductor 2 to conductor 1, work will be done externally, since at any stage conductor 1 is at a higher potential than conductor 2. To calculate the total work done, we first calculate the work done in a small step involving transfer of an infinitesimal (i.e., vanishingly small) amount of charge. Consider the intermediate situation when the conductors 1 and 2 have charges Q' and $-Q'$ respectively. At this stage, the potential difference V' between conductors 1 to 2 is Q'/C, where C is the capacitance of the system. Next imagine that a small charge $\delta Q'$ is transferred from conductor 2 to 1. Work done in this step (δW), resulting in charge Q' on conductor 1 increasing to $Q' + \delta Q'$, is given by

$$\delta W = V'\delta Q' = \frac{Q'}{C}\delta Q' \qquad (2.68)$$