

Table 10.2 The viscosities of some fluids

| Fluid | T(°C) | Viscosity (mPl) |
|-------------|-------|-----------------|
| Water | 20 | 1.0 |
| | 100 | 0.3 |
| Blood | 37 | 2.7 |
| | 16 | 113 |
| Machine Oil | 38 | 34 |
| | 20 | 830 |
| Glycerine | 20 | 830 |
| Honey | – | 200 |
| Air | 0 | 0.017 |
| | 40 | 0.019 |

10.5.1 Stokes' Law

When a body falls through a fluid it drags the layer of the fluid in contact with it. A relative motion between the different layers of the fluid is set and, as a result, the body experiences a retarding force. Falling of a raindrop and swinging of a pendulum bob are some common examples of such motion. It is seen that the viscous force is proportional to the velocity of the object and is opposite to the direction of motion. The other quantities on which the force F depends are viscosity η of the fluid and radius a of the sphere. Sir George G. Stokes (1819–1903), an English scientist enunciated clearly the viscous drag force F as

$$F = 6\pi\eta av \quad (10.19)$$

This is known as Stokes' law. We shall not derive Stokes' law.

This law is an interesting example of retarding force, which is proportional to velocity. We can study its consequences on an object falling through a viscous medium. We consider a raindrop in air. It accelerates initially due to gravity. As the velocity increases, the retarding force also increases. Finally, when viscous force plus buoyant force becomes equal to the force due to gravity, the net force becomes zero and so does the acceleration. The sphere (raindrop) then descends with a constant velocity. Thus, in equilibrium, this terminal velocity v_t is given by

$$6\pi\eta av_t = (4\pi/3) a^3 (\rho - \sigma)g$$

where ρ and σ are mass densities of sphere and the fluid, respectively. We obtain

$$v_t = 2a^2 (\rho - \sigma)g / (9\eta) \quad (10.20)$$

So the terminal velocity v_t depends on the square of the radius of the sphere and inversely on the viscosity of the medium.

You may like to refer back to Example 6.2 in this context.

► **Example 10.10** The terminal velocity of a copper ball of radius 2.0 mm falling through a tank of oil at 20°C is 6.5 cm s⁻¹. Compute the viscosity of the oil at 20°C. Density of oil is 1.5 × 10³ kg m⁻³, density of copper is 8.9 × 10³ kg m⁻³.

Answer We have $v_t = 6.5 \times 10^{-2} \text{ ms}^{-1}$, $a = 2 \times 10^{-3} \text{ m}$, $g = 9.8 \text{ ms}^{-2}$, $\rho = 8.9 \times 10^3 \text{ kg m}^{-3}$,

$\sigma = 1.5 \times 10^3 \text{ kg m}^{-3}$. From Eq. (10.20)

$$\begin{aligned} \eta &= \frac{2}{9} \times \frac{(2 \times 10^{-3})^2 \text{ m}^2 \times 9.8 \text{ ms}^{-2}}{6.5 \times 10^{-2} \text{ ms}^{-1}} \times 7.4 \times 10^3 \text{ kg m}^{-3} \\ &= 9.9 \times 10^{-1} \text{ kg m}^{-1} \text{ s}^{-1} \end{aligned}$$

10.6 SURFACE TENSION

You must have noticed that, oil and water do not mix; water wets you and me but not ducks; mercury does not wet glass but water sticks to it, oil rises up a cotton wick, inspite of gravity, Sap and water rise up to the top of the leaves of the tree, hair of a paint brush do not cling together when dry and even when dipped in water but form a fine tip when taken out of it. All these and many more such experiences are related with the free surfaces of liquids. As liquids have no definite shape but have a definite volume, they acquire a free surface when poured in a container. These surfaces possess some additional energy. This phenomenon is known as surface tension and it is concerned with only liquid as gases do not have free surfaces. Let us now understand this phenomena.

10.6.1 Surface Energy

A liquid stays together because of attraction between molecules. Consider a molecule well inside a liquid. The intermolecular distances are such that it is attracted to all the surrounding molecules [Fig. 10.16(a)]. This attraction results in a negative potential energy for the molecule, which depends on the number and distribution of molecules around the chosen one. But the average potential energy of all the molecules is the same. This is supported by the fact that to take a collection of such molecules (the liquid)

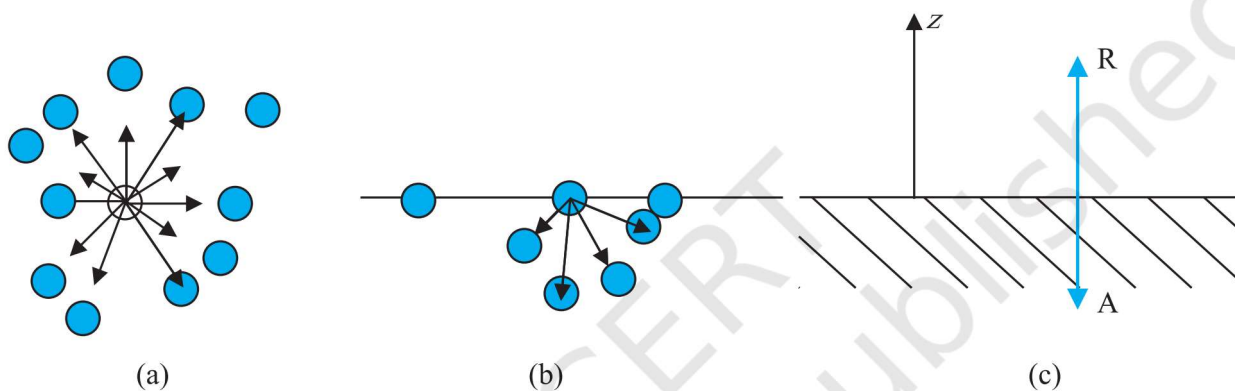


Fig. 10.16 Schematic picture of molecules in a liquid, at the surface and balance of forces. (a) Molecule inside a liquid. Forces on a molecule due to others are shown. Direction of arrows indicates attraction of repulsion. (b) Same, for a molecule at a surface. (c) Balance of attractive (A) and repulsive (R) forces.

and to disperse them far away from each other in order to evaporate or vaporise, the heat of evaporation required is quite large. For water it is of the order of 40 kJ/mol.

Let us consider a molecule near the surface Fig. 10.16(b). Only lower half side of it is surrounded by liquid molecules. There is some negative potential energy due to these, but obviously it is less than that of a molecule in bulk, i.e., the one fully inside. Approximately it is half of the latter. Thus, molecules on a liquid surface have some extra energy in comparison to molecules in the interior. A liquid, thus, tends to have the least surface area which external conditions permit. Increasing surface area requires energy. Most surface phenomenon can be understood in

terms of this fact. What is the energy required for having a molecule at the surface? As mentioned above, roughly it is half the energy required to remove it entirely from the liquid i.e., half the heat of evaporation.

Finally, what is a surface? Since a liquid consists of molecules moving about, there cannot be a perfectly sharp surface. The density of the liquid molecules drops rapidly to zero around $z = 0$ as we move along the direction indicated Fig 10.16 (c) in a distance of the order of a few molecular sizes.

10.6.2 Surface Energy and Surface Tension

As we have discussed that an extra energy is associated with surface of liquids, the creation of more surface (spreading of surface) keeping other things like volume fixed requires additional energy. To appreciate this, consider a horizontal liquid film ending in bar free to slide over parallel guides Fig (10.17).

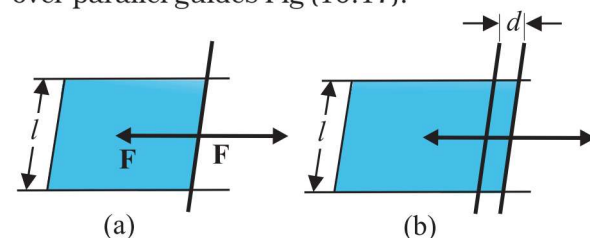


Fig. 10.17 Stretching a film. (a) A film in equilibrium; (b) The film stretched an extra distance.

Suppose that we move the bar by a small distance d as shown. Since the area of the surface increases, the system now has more energy, this means that some work has been done against an internal force. Let this internal force be \mathbf{F} , the work done by the applied force is $\mathbf{F} \cdot \mathbf{d} = Fd$. From conservation of energy, this is stored as additional energy in the film. If the surface energy of the film is S per unit area, the extra area is $2dl$. A film has two sides and the liquid in between, so there are two surfaces and the extra energy is

$$S(2dl) = Fd \quad (10.21)$$

$$\text{Or, } S = Fd/2dl = F/2l \quad (10.22)$$

This quantity S is the magnitude of surface tension. It is equal to the surface energy per unit area of the liquid interface and is also equal to the force per unit length exerted by the fluid on the movable bar.

So far we have talked about the surface of one liquid. More generally, we need to consider fluid surface in contact with other fluids or solid surfaces. The surface energy in that case depends on the materials on both sides of the surface. For example, if the molecules of the materials attract each other, surface energy is reduced while if they repel each other the surface energy is increased. Thus, more appropriately, the surface energy is the energy of the interface between two materials and depends on both of them.

We make the following observations from above:

- (i) Surface tension is a force per unit length (or surface energy per unit area) acting in the plane of the interface between the plane of the liquid and any other substance; it also is the extra energy that the molecules at the interface have as compared to molecules in the interior.
- (ii) At any point on the interface besides the boundary, we can draw a line and imagine

equal and opposite surface tension forces S per unit length of the line acting perpendicular to the line, in the plane of the interface. The line is in equilibrium. To be more specific, imagine a line of atoms or molecules at the surface. The atoms to the left pull the line towards them; those to the right pull it towards them! This line of atoms is in equilibrium under tension. If the line really marks the end of the interface, as in Figure 10.16 (a) and (b) there is only the force S per unit length acting inwards.

Table 10.3 gives the surface tension of various liquids. The value of surface tension depends on temperature. Like viscosity, the surface tension of a liquid usually falls with temperature.

Table 10.3 Surface tension of some liquids at the temperatures indicated with the heats of the vaporisation

| Liquid | Temp (°C) | Surface Tension (N/m) | Heat of vaporisation (kJ/mol) |
|---------|-----------|-----------------------|-------------------------------|
| Helium | -270 | 0.000239 | 0.115 |
| Oxygen | -183 | 0.0132 | 7.1 |
| Ethanol | 20 | 0.0227 | 40.6 |
| Water | 20 | 0.0727 | 44.16 |
| Mercury | 20 | 0.4355 | 63.2 |

A fluid will stick to a solid surface if the surface energy between fluid and the solid is smaller than the sum of surface energies between solid-air, and fluid-air. Now there is attraction between the solid surface and the liquid. It can be directly measured experimentally as schematically shown in Fig. 10.18. A flat vertical glass plate, below which a vessel of some liquid is kept, forms one arm of the balance. The plate is balanced by weights

on the other side, with its horizontal edge just over water. The vessel is raised slightly till the liquid just touches the glass plate and pulls it down a little because of surface tension. Weights are added till the plate just clears water.

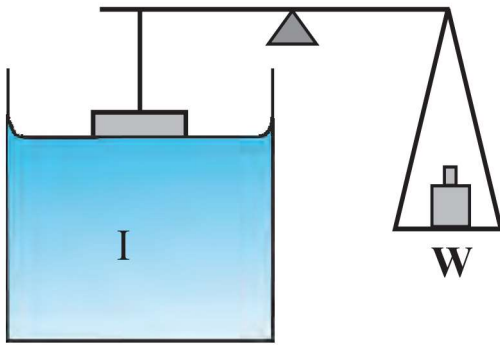


Fig. 10.18 Measuring Surface Tension.

Suppose the additional weight required is W . Then from Eq. 10.22 and the discussion given there, the surface tension of the liquid-air interface is

$$S_{la} = (W/2l) = (mg/2l) \tag{10.23}$$

where m is the extra mass and l is the length of the plate edge. The subscript (la) emphasises the fact that the liquid-air interface tension is involved.

10.6.3 Angle of Contact

The surface of liquid near the plane of contact, with another medium is in general curved. The angle between tangent to the liquid surface at the point of contact and solid surface inside the liquid is termed as angle of contact. It is denoted by θ . It is different at interfaces of different pairs of liquids and solids. The value of θ determines whether a liquid will spread on the surface of a solid or it will form droplets on it. For example, water forms droplets on lotus leaf as shown in Fig. 10.19 (a) while spreads over a clean plastic plate as shown in Fig. 10.19(b).

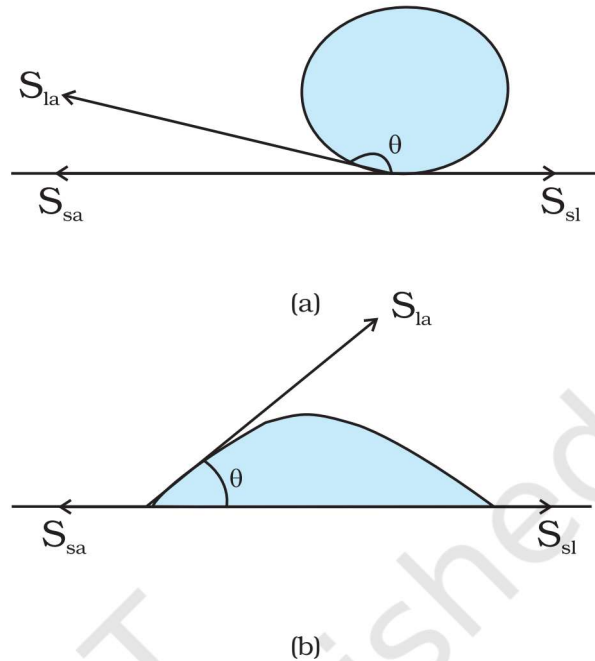


Fig. 10.19 Different shapes of water drops with interfacial tensions (a) on a lotus leaf (b) on a clean plastic plate.

We consider the three interfacial tensions at all the three interfaces, liquid-air, solid-air and solid-liquid denoted by S_{la} , S_{sa} and S_{sl} , respectively as given in Fig. 10.19 (a) and (b). At the line of contact, the surface forces between the three media must be in equilibrium. From the Fig. 10.19(b) the following relation is easily derived.

$$S_{la} \cos \theta + S_{sl} = S_{sa} \tag{10.24}$$

The angle of contact is an obtuse angle if $S_{sl} > S_{la}$ as in the case of water-leaf interface while it is an acute angle if $S_{sl} < S_{la}$ as in the case of water-plastic interface. When θ is an obtuse angle then molecules of liquids are attracted strongly to themselves and weakly to those of solid, it costs a lot of energy to create a liquid-solid surface, and liquid then does not wet the solid. This is what happens with water on a waxy or oily surface, and with mercury on any surface. On the other hand, if the molecules of the liquid are strongly attracted to those of

the solid, this will reduce S_{sl} and therefore, $\cos \theta$ may increase or θ may decrease. In this case θ is an acute angle. This is what happens for water on glass or on plastic and for kerosene oil on virtually anything (it just spreads). Soaps, detergents and dyeing substances are wetting agents. When they are added the angle of contact becomes small so that these may penetrate well and become effective. Water proofing agents on the other hand are added to create a large angle of contact between the water and fibres.

10.6.4 Drops and Bubbles

One consequence of surface tension is that free liquid drops and bubbles are spherical if effects of gravity can be neglected. You must have seen this especially clearly in small drops just formed in a high-speed spray or jet, and in soap bubbles blown by most of us in childhood. Why are drops and bubbles spherical? What keeps soap bubbles stable?

As we have been saying repeatedly, a liquid-air interface has energy, so for a given volume the surface with minimum energy is the one with the least area. The sphere has this property. Though it is out of the scope of this book, but you can check that a sphere is better than at least a cube in this respect! So, if gravity and other forces (e.g. air resistance) were ineffective, liquid drops would be spherical.

Another interesting consequence of surface tension is that the pressure inside a spherical drop Fig. 10.20(a) is more than the pressure outside. Suppose a spherical drop of radius r is in equilibrium. If its radius increase by Δr . The extra surface energy is

$$[4\pi(r + \Delta r)^2 - 4\pi r^2] S_{la} = 8\pi r \Delta r S_{la} \quad (10.25)$$

If the drop is in equilibrium this energy cost is balanced by the energy gain due to expansion under the pressure difference ($P_i - P_o$) between the inside of the bubble and the outside. The work done is

$$W = (P_i - P_o) 4\pi r^2 \Delta r \quad (10.26)$$

so that

$$(P_i - P_o) = (2 S_{la} / r) \quad (10.27)$$

In general, for a liquid-gas interface, the convex side has a higher pressure than the concave side. For example, an air bubble in a liquid, would have higher pressure inside it. See Fig 10.20 (b).

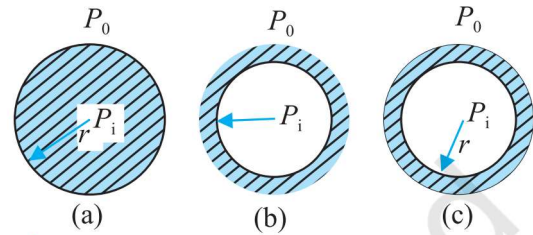


Fig. 10.20 Drop, cavity and bubble of radius r .

A bubble Fig 10.20 (c) differs from a drop and a cavity; in this it has two interfaces. Applying the above argument we have for a bubble

$$(P_i - P_o) = (4 S_{la} / r) \quad (10.28)$$

This is probably why you have to blow hard, but not too hard, to form a soap bubble. A little extra air pressure is needed inside!

10.6.5 Capillary Rise

One consequence of the pressure difference across a curved liquid-air interface is the well-known effect that water rises up in a narrow tube in spite of gravity. The word capilla means hair in Latin; if the tube were hair thin, the rise would be very large. To see this, consider a vertical capillary tube of circular cross section (radius a) inserted into an open vessel of water (Fig. 10.21). The contact angle between water

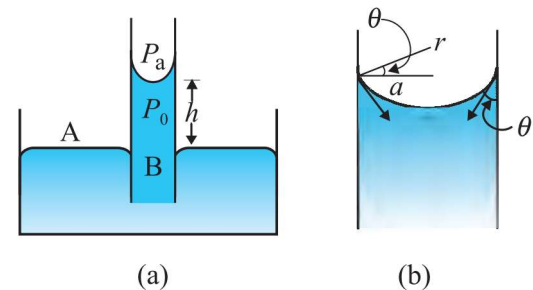


Fig. 10.21 Capillary rise, (a) Schematic picture of a narrow tube immersed water. (b) Enlarged picture near interface.

and glass is acute. Thus the surface of water in the capillary is concave. This means that there is a pressure difference between the two sides of the top surface. This is given by

$$\begin{aligned}(P_i - P_o) &= (2S/r) = 2S/(a \sec \theta) \\ &= (2S/a) \cos \theta\end{aligned}\quad (10.29)$$

Thus the pressure of the water inside the tube, just at the meniscus (air-water interface) is less than the atmospheric pressure. Consider the two points A and B in Fig. 10.21(a). They must be at the same pressure, namely

$$P_o + h \rho g = P_i = P_A \quad (10.30)$$

where ρ is the density of water and h is called the capillary rise [Fig. 10.21(a)]. Using Eq. (10.29) and (10.30) we have

$$h \rho g = (P_i - P_o) = (2S \cos \theta)/a \quad (10.31)$$

The discussion here, and the Eqs. (10.26) and (10.27) make it clear that the capillary rise is due to surface tension. It is larger, for a smaller a . Typically it is of the order of a few cm for fine capillaries. For example, if $a = 0.05$ cm, using the value of surface tension for water (Table 10.3), we find that

$$\begin{aligned}h &= 2S/(\rho g a) \\ &= \frac{2 \times (0.073 \text{ N m}^{-1})}{(10^3 \text{ kg m}^{-3})(9.8 \text{ m s}^{-2})(5 \times 10^{-4} \text{ m})} \\ &= 2.98 \times 10^{-2} \text{ m} = 2.98 \text{ cm}\end{aligned}$$

Notice that if the liquid meniscus is convex, as for mercury, i.e., if $\cos \theta$ is negative then from Eq. (10.30) for example, it is clear that the liquid will be lower in the capillary!

10.6.6 Detergents and Surface Tension

We clean dirty clothes containing grease and oil stains sticking to cotton or other fabrics by adding detergents or soap to water, soaking clothes in it and shaking. Let us understand this process better.

Washing with water does not remove grease stains. This is because water does not wet greasy dirt; i.e., there is very little area of contact between them. If water could wet grease, the flow of water could carry some grease away. Something of this sort is achieved through detergents. The molecules of detergents are

hairpin shaped, with one end attracted to water and the other to molecules of grease, oil or wax, thus tending to form water-oil interfaces. The result is shown in Fig. 10.22 as a sequence of figures.

In our language, we would say that addition of detergents, whose molecules attract at one end and say, oil on the other, reduces drastically the surface tension S (water-oil). It may even become energetically favourable to form such interfaces, i.e., globs of dirt surrounded by detergents and then by water. This kind of process using surface active detergents or surfactants is important not only for cleaning, but also in recovering oil, mineral ores etc.

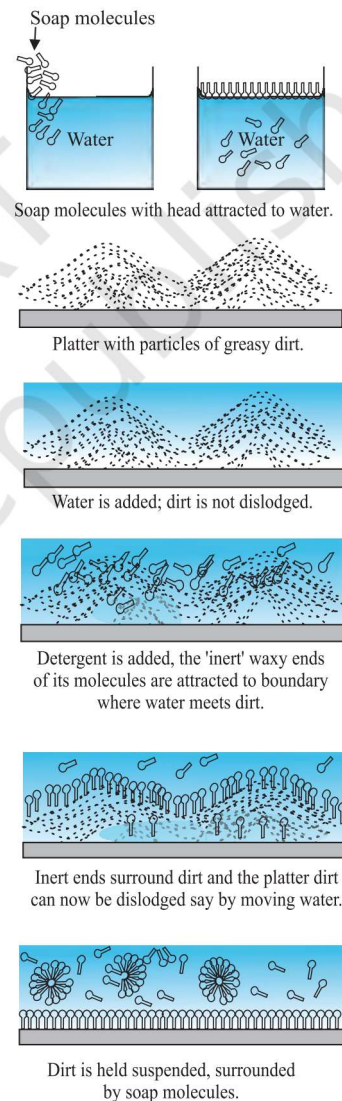


Fig. 10.22 Detergent action in terms of what detergent molecules do.

► **Example 10.11** The lower end of a capillary tube of diameter 2.00 mm is dipped 8.00 cm below the surface of water in a beaker. What is the pressure required in the tube in order to blow a hemispherical bubble at its end in water? The surface tension of water at temperature of the experiments is $7.30 \times 10^{-2} \text{ Nm}^{-1}$. 1 atmospheric pressure = $1.01 \times 10^5 \text{ Pa}$, density of water = 1000 kg/m^3 , $g = 9.80 \text{ m s}^{-2}$. Also calculate the excess pressure.

Answer The excess pressure in a bubble of gas in a liquid is given by $2S/r$, where S is the surface tension of the liquid-gas interface. You should note there is only one liquid surface in this case. (For a bubble of liquid in a gas, there are two liquid surfaces, so the formula for

excess pressure in that case is $4S/r$.) The radius of the bubble is r . Now the pressure outside the bubble P_o equals atmospheric pressure plus the pressure due to 8.00 cm of water column. That is

$$P_o = (1.01 \times 10^5 \text{ Pa} + 0.08 \text{ m} \times 1000 \text{ kg m}^{-3} \times 9.80 \text{ m s}^{-2}) \\ = 1.01784 \times 10^5 \text{ Pa}$$

Therefore, the pressure inside the bubble is

$$P_i = P_o + 2S/r \\ = 1.01784 \times 10^5 \text{ Pa} + (2 \times 7.3 \times 10^{-2} \text{ Pa m} / 10^{-3} \text{ m}) \\ = (1.01784 + 0.00146) \times 10^5 \text{ Pa} \\ = 1.02 \times 10^5 \text{ Pa}$$

where the radius of the bubble is taken to be equal to the radius of the capillary tube, since the bubble is hemispherical! (The answer has been rounded off to three significant figures.) The excess pressure in the bubble is 146 Pa. ◀

SUMMARY

1. The basic property of a fluid is that it can flow. The fluid does not have any resistance to change of its shape. Thus, the shape of a fluid is governed by the shape of its container.
2. A liquid is incompressible and has a free surface of its own. A gas is compressible and it expands to occupy all the space available to it.
3. If F is the normal force exerted by a fluid on an area A then the average pressure P_{av} is defined as the ratio of the force to area

$$P_{av} = \frac{F}{A}$$

4. The unit of the pressure is the pascal (Pa). It is the same as N m^{-2} . Other common units of pressure are
 $1 \text{ atm} = 1.01 \times 10^5 \text{ Pa}$
 $1 \text{ bar} = 10^5 \text{ Pa}$
 $1 \text{ torr} = 133 \text{ Pa} = 0.133 \text{ kPa}$
 $1 \text{ mm of Hg} = 1 \text{ torr} = 133 \text{ Pa}$
5. *Pascal's law* states that: Pressure in a fluid at rest is same at all points which are at the same height. A change in pressure applied to an enclosed fluid is transmitted undiminished to every point of the fluid and the walls of the containing vessel.
6. The pressure in a fluid varies with depth h according to the expression
 $P = P_a + \rho gh$
 where ρ is the density of the fluid, assumed uniform.
7. The volume of an incompressible fluid passing any point every second in a pipe of non uniform cross-section is the same in the steady flow.
 $vA = \text{constant}$ (v is the velocity and A is the area of cross-section)
 The equation is due to mass conservation in incompressible fluid flow.