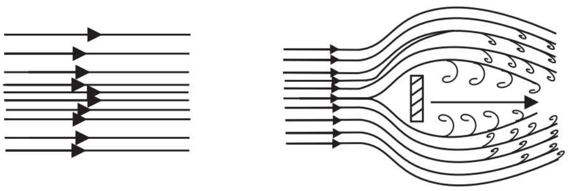
stream encounters rocks, small foamy whirlpool-like regions called 'white water rapids are formed.

Figure 10.8 displays streamlines for some typical flows. For example, Fig. 10.8(a) describes a laminar flow where the velocities at different points in the fluid may have different magnitudes but their directions are parallel. Figure 10.8 (b) gives a sketch of turbulent flow.



(a) (b)

Fig. 10.8 *(a) Some streamlines for fluid flow. (b) A jet of air striking a flat plate placed perpendicular to it. This is an example of turbulent flow.*

10.4 BERNOULLI'S PRINCIPLE

Fluid flow is a complex phenomenon. But we can obtain some useful properties for steady or streamline flows using the conservation of energy.

Consider a fluid moving in a pipe of varying cross-sectional area. Let the pipe be at varying heights as shown in Fig. 10.9. We now suppose that an incompressible fluid is flowing through the pipe in a steady flow. Its velocity must change as a consequence of equation of continuity. A force is required to produce this acceleration, which is caused by the fluid surrounding it, the pressure must be different in different regions. Bernoulli's equation is a general expression that relates the pressure difference between two points in a pipe to both velocity changes (kinetic energy change) and elevation (height) changes (potential energy change). The Swiss Physicist Daniel Bernoulli developed this relationship in 1738.

Consider the flow at two regions 1 (i.e., BC) and 2 (i.e., DE). Consider the fluid initially lying between B and D. In an infinitesimal time interval Δt, this fluid would have moved. Suppose v_1 is the speed at B and v_2 at D, then fluid initially at B has moved a distance $v_1\Delta t$ to C ($v_1\Delta t$ is small enough to assume constant cross-section along BC). In the same interval Δt the fluid initially at D moves to E, a distance equal to $v_2\Delta t$. Pressures P_1 and P_2 act as shown on the plane faces of areas A_1 and A_2 binding the two regions. The work done on the fluid at left end (BC) is $W_1 = P_1A_1(v_1\Delta t) = P_1\Delta V$. Since the same volume ΔV passes through both the regions (from the equation of continuity) the work done by the fluid at the other end (DE) is $W_2 = P_2A_2(v_2\Delta t) = P_2\Delta V$ or, the work done on the fluid is $-P_2\Delta V$. So the total work done on the fluid is

$$W_1 - W_2 = (P_1 - P_2)\,\Delta V$$

Part of this work goes into changing the kinetic energy of the fluid, and part goes into changing the gravitational potential energy. If the density of the fluid is ρ and $\Delta m = \rho A_1 v_1 \Delta t = \rho\Delta V$ is the mass passing through the pipe in time Δt, then change in gravitational potential energy is

$$\Delta U = \rho g \Delta V\,(h_2 - h_1)$$

The change in its kinetic energy is

$$\Delta K = \quad \rho\,\Delta V\,(v_2^2 - v_1^2)$$

We can employ the work – energy theorem (Chapter 6) to this volume of the fluid and this yields

$$(P_1 - P_2)\,\Delta V = \left(\frac{1}{2}\right)\rho\,\Delta V\,(v_2^2 - v_1^2) + \rho g \Delta V\,(h_2 - h_1)$$

We now divide each term by ΔV to obtain

$$(P_1 - P_2) = \left(\frac{1}{2}\right)\rho\,(v_2^2 - v_1^2) + \rho g\,(h_2 - h_1)$$

Daniel Bernoulli (1700–1782)

Daniel Bernoulli was a Swiss scientist and mathematician, who along with Leonard Euler had the distinction of winning the French Academy prize for mathematics 10 times. He also studied medicine and served as a professor of anatomy and botany for a while at Basle, Switzerland. His most well-known work was in hydrodynamics, a subject he developed from a single principle: the conservation of energy. His work included calculus, probability, the theory of vibrating strings, and applied mathematics. He has been called the founder of mathematical physics.

We can rearrange the above terms to obtain

$$P_1 + \left(\frac{1}{2}\right) \rho v_1^{\,2} + \rho g h_1 = P_2 + \left(\frac{1}{2}\right) \rho v_2^{\,2} + \rho g h_2 \tag{10.12}$$

This is **Bernoulli's equation**. Since 1 and 2 refer to any two locations along the pipeline, we may write the expression in general as

$$P + \left(\frac{1}{2}\right) \rho v^2 + \rho g h = \text{constant} \tag{10.13}$$

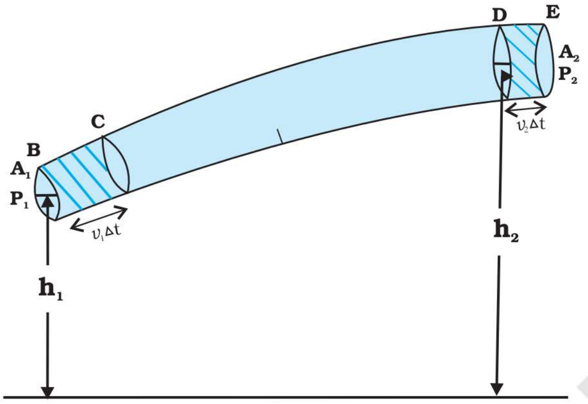


Fig. 10.9 *The flow of an ideal fluid in a pipe of varying cross section. The fluid in a section of length $v_1\Delta t$ moves to the section of length $v_2\Delta t$ in time Δt.*

In words, the Bernoulli's relation may be stated as follows: As we move along a streamline the sum of the pressure (P), the kinetic energy per unit volume $\left(\frac{\rho v^2}{2}\right)$ and the potential energy per unit volume ($\rho g h$) remains a constant.

Note that in applying the energy conservation principle, there is an assumption that no energy is lost due to friction. But in fact, when fluids flow, some energy does get lost due to internal friction. This arises due to the fact that in a fluid flow, the different layers of the fluid flow with different velocities. These layers exert frictional forces on each other resulting in a loss of energy. This property of the fluid is called viscosity and is discussed in more detail in a later section. The lost kinetic energy of the fluid gets converted into heat energy. Thus, Bernoulli's equation ideally applies to fluids with zero viscosity or non-viscous fluids. Another restriction on application of Bernoulli theorem is that the fluids must be incompressible, as the elastic energy of the fluid is also not taken into consideration. In practice, it has a large number of useful applications and can help explain a wide variety of phenomena for low viscosity incompressible fluids. Bernoulli's equation also does not hold for non-steady or turbulent flows, because in that situation velocity and pressure are constantly fluctuating in time.

When a fluid is at rest i.e., its velocity is zero everywhere, Bernoulli's equation becomes

$$P_1 + \rho g h_1 = P_2 + \rho g h_2$$

$$(P_1 - P_2) = \rho g\,(h_2 - h_1)$$

which is same as Eq. (10.6).

10.4.1 Speed of Efflux: Torricelli's Law

The word efflux means fluid outflow. Torricelli discovered that the speed of efflux from an open tank is given by a formula identical to that of a freely falling body. Consider a tank containing a liquid of density ρ with a small hole in its side at a height y_1 from the bottom (see Fig. 10.10). The air above the liquid, whose surface is at height y_2, is at pressure P. From the equation of continuity [Eq. (10.10)] we have

$$v_1 A_1 = v_2 A_2$$

$$v_2 = \frac{A_1}{A_2} v_1$$

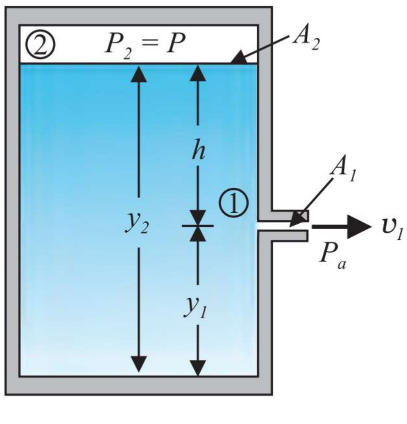


Fig. 10.10 *Torricelli's law. The speed of efflux, v_1, from the side of the container is given by the application of Bernoulli's equation. If the container is open at the top to the atmosphere then $v_1 = \sqrt{2\,g\,h}$.*

If the cross-sectional area of the tank A_2 is much larger than that of the hole ($A_2 >> A_1$), then we may take the fluid to be approximately at rest at the top, i.e., $v_2 = 0$. Now, applying the Bernoulli equation at points 1 and 2 and noting that at the hole $P_1 = P_a$, the atmospheric pressure, we have from Eq. (10.12)

$$P_a + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P + \rho g y_2$$

Taking $y_2 - y_1 = h$ we have

$$v_1 = \sqrt{2g\,h + \frac{2(P - P_a)}{\rho}} \qquad (10.14)$$

When $P >> P_a$ and $2\,g\,h$ may be ignored, the speed of efflux is determined by the container pressure. Such a situation occurs in rocket propulsion. On the other hand, if the tank is open to the atmosphere, then $P = P_a$ and

$$v_1 = \sqrt{2gh} \qquad (10.15)$$

This is also the speed of a freely falling body. Equation (10.15) represents **Torricelli's law**.

10.4.2 Venturi-meter

The Venturi-meter is a device to measure the flow speed of incompressible fluid. It consists of a tube with a broad diameter and a small constriction at the middle as shown in Fig. (10.11). A manometer in the form of a U-tube is also attached to it, with one arm at the broad neck point of the tube and the other at constriction as shown in Fig. (10.11). The manometer contains a liquid of density ρ_m. The speed v_1 of the liquid flowing through the tube at the broad neck area A is to be measured from equation of continuity Eq. (10.10) the speed at the constriction becomes $v_2 = \frac{A}{a}v_1$. Then using Bernoulli's equation (Eq.10.12) for $(h_1=h_2)$, we get

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_1^2 (A/a)^2$$

So that

$$P_1 - P_2 = \frac{1}{2}\rho v_1^2 \left[\left(\frac{A}{a}\right)^2 - 1\right] \qquad (10.16)$$

This pressure difference causes the fluid in the U-tube connected at the narrow neck to rise in comparison to the other arm. The difference in height h measure the pressure difference.

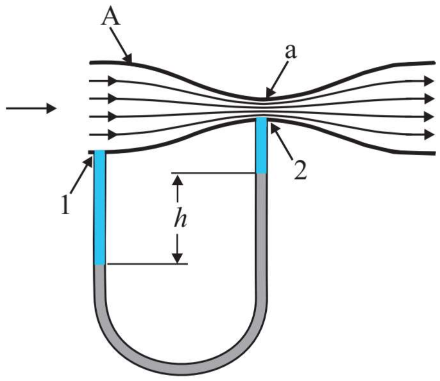


Fig. 10.11 *A schematic diagram of Venturi-meter.*

$$P_1 - P_2 = \rho_m g h = \frac{1}{2}\rho v_1^2 \left[\left(\frac{A}{a}\right)^2 - 1\right]$$

So that the speed of fluid at wide neck is

$$v_1 = \sqrt{\left(\frac{2\rho_m gh}{\rho}\right)}\left(\left(\frac{A}{a}\right)^2 - 1\right)^{-1/2} \qquad (10.17)$$

The principle behind this meter has many applications. The carburetor of automobile has a Venturi channel (nozzle) through which air flows with a high speed. The pressure is then lowered at the narrow neck and the petrol (gasoline) is sucked up in the chamber to provide the correct mixture of air to fuel necessary for combustion. Filter pumps or aspirators, Bunsen burner, atomisers and sprayers [See Fig. 10.12] used for perfumes or to spray insecticides work on the same principle.

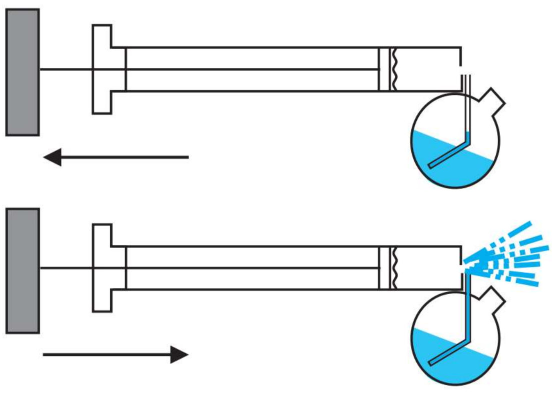


Fig. 10.12 *The spray gun. Piston forces air at high speeds causing a lowering of pressure at the neck of the container.*