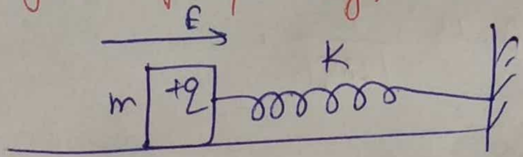
(1) A wooden block performs SHM on a frictionless surface with frequency ν_0. the block contains a +ve charge q on its surface. If now, an uniform electric field $\vec{E}$ switched on as shown, then SHM of block will be:

(a) ✓ of the same frequency and with shifted mean position.

b) of the same frequency and with same mean position.

c) of changed frequency and with shifted mean position.

d) of changed frequency with same mean position.

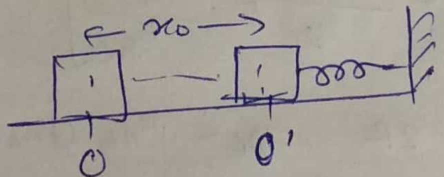


Solution:-

Before $\vec{E}$ is switched on,

$$\nu_0 = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$$

when $\vec{E}$ is switched on then block shifts from its p.s.



block is in equilibrium at O'

$$\therefore \; qE = Kx_0 \Rightarrow x_0 = \frac{qE}{K}$$

Now equation of motion ⇒

$$m\frac{d^2x}{dt^2} = -K(x+x_0) + qE$$

$$= -Kx - \cancel{Kx_0} + \cancel{qE}$$

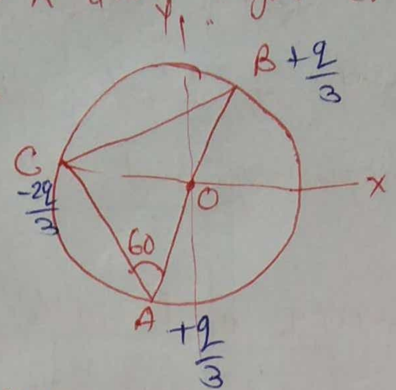
$\because qE = Kx_0$

$$\boxed{\frac{d^2x}{dt^2} = -\underbrace{\frac{K}{m}}_{\omega^2}x} \Rightarrow \boxed{\omega = \sqrt{\frac{K}{m}}} \rightarrow \boxed{\nu = \frac{1}{2\pi}\sqrt{\frac{K}{m}}}$$

frequency not change.

Hence option (a) is correct.

(2) Consider a system of three charges $\frac{q}{3}$, $\frac{q}{3}$ & $-\frac{2q}{3}$ placed at points A, B and C resp. Take O to be centre of the circle of radius R and angle CAB = 60°.



A) the electric field at point O is $\frac{1}{8\pi\epsilon_0 r}$ directed along the -ve x-axis.

B) the potential energy of the system is zero.

C) then magnitude of force b/w the charge C & B is $\frac{q^2}{54\pi\epsilon R^2}$.

d) the potential at point O is $\frac{q}{12\pi\epsilon_0 R}$.

Solution:- $q_A = \frac{q}{3}$, $q_B = \frac{q}{3}$, $q_C = -\frac{2q}{3}$

$$\vec{E_0} = \frac{-2q/3}{4\pi\epsilon R^2}\hat{x} = \frac{-q^2}{6\pi\epsilon R^2}\hat{x}$$ ⇒ So option (A ✗)

$$U = \frac{1}{4\pi\epsilon_0}\left[\frac{q_A q_B}{r_{AB}} + \frac{q_C q_A}{r_{AC}} + \frac{q_B q_C}{r_{BC}}\right]$$

Now $r_{AB} = 2R$, $r_{AC} = r_{AB}\sin 30° = \frac{1}{2} r_{AB}$

$r_{BC} = \sqrt{3}R$

$$\therefore U = \frac{1}{4\pi\epsilon}\left[\frac{q^2}{18R} - \frac{2q^2}{9R} - \frac{2q^2}{9\sqrt{3}R}\right] \neq 0 \Rightarrow$$ (B ✗)

$$F_{BC} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_B q_C}{r_{BC}^2} = \frac{q^2}{54\pi\epsilon_0 R^2}$$ ⇒ (C ✓)

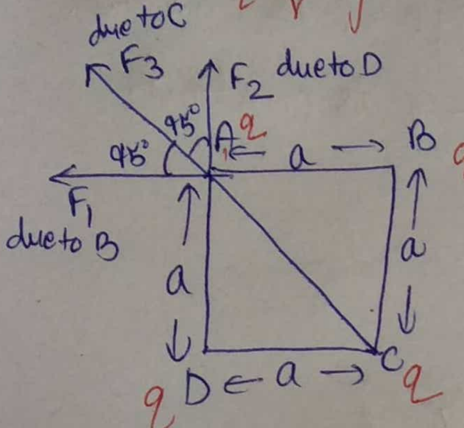
$$V_O = \frac{1}{4\pi\epsilon_0}\left[\frac{q_A}{R} + \frac{q_B}{R} + \frac{q_C}{R}\right] = 0$$ ⇒ (D ✗)

at point O

$\boxed{\text{correct option (C)}}$

③ Four point charges, each of +q are rigidly fixed at four corners of a square planar soap film of side a. The surface tension of soap film is γ. The system of charges and planar film are in equilibrium and

$a = k\left[\frac{q^2}{\gamma}\right]^{1/N}$, N is ______ ? $K = \left(\frac{1}{4\pi\varepsilon_0}\right)$



the force on line BC due to surface tension is γa.
↓
constant

Now $F_1 = \frac{q^2}{4\pi\varepsilon_0 a^2}$, $F_2 = \frac{q^2}{4\pi\varepsilon_0 a^2}$

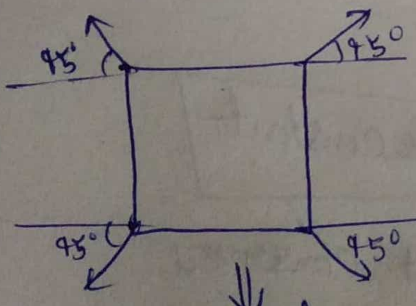
$F_3 = \frac{q^2}{4\pi\varepsilon_0(\sqrt{2}a)^2} = \frac{q^2}{8\pi\varepsilon_0 a^2}$

∴ resultant force at point A

$= F_1\cos 45^\circ + F_2\cos 45^\circ + F_3$

$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{a^2}\left[\sqrt{2}+\frac{1}{2}\right]$

$= \frac{Kq^2}{a^2}\left[\sqrt{2}+\frac{1}{2}\right]$



⇓

total force on the wire $= 2F\cos 45^\circ$

Now force is equal to γa.

$\gamma a = 2\frac{Kq^2}{a^2}\left[\sqrt{2}+\frac{1}{2}\right]\frac{1}{\sqrt{2}}$

$a^3 = \sqrt{2}K\left(\sqrt{2}+\frac{1}{2}\right)\frac{q^2}{\gamma}$

$\boxed{N=3}$

Ans ← $\boxed{a = K\left(\frac{q^2}{\gamma}\right)^{1/3}}$

(4) Under the influence of the coulomb field of charge +Q, a charge −q is moving around it in an elliptical orbit. Find out the correct statement(s).

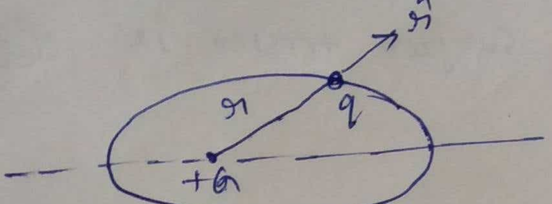
(A) The angular momentum of the charge −q is constant.

(B) The linear momentum of charge −q is constant.

(C) The angular velocity of the charge −q is constant.

(d) The linear speed of the charge −q is constant.

Solution:—



Torque on charge −q due to coulomb force

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$= \vec{r} \times F\hat{r} \quad (\because \vec{r} \ \& \ \vec{F} \text{ are parallel} \quad \sin 0^\circ = 0)$$

$$\vec{\tau} = 0$$

$$\Rightarrow \frac{d\vec{L}}{dt} = 0 \Rightarrow \boxed{\vec{L} = \text{constant}}$$

but $\vec{F} = \frac{1}{4\pi\epsilon_0} \cdot \frac{qQ}{r^2} \hat{r} \neq 0$

$$\Rightarrow \frac{d\vec{p}}{dt} \neq 0 \Rightarrow \boxed{\vec{p} \neq \text{constant}}$$

p is not conserved.

$L = m\omega r^2$

as L is constant ω must vary

as $\cancel{\omega = vr}$ $V = \omega r$

$\Rightarrow$ Correct option $\Rightarrow$ C

(5) A particle of mass 10^{-1} kg and charge 1.0 C is initially at rest. At time $t=0$, the particle comes under the influence of an electric field $\vec{E}(t) = E_0 \sin\omega t$ where $E_0 = 1.0$ N/C and $\omega = 10^3$ rad/s. Consider the effect of only the electric force on the particle. Then the maximum speed, in m/s, obtained by the particle at subsequent time is:-

Solution:-

$$\vec{F} = q\vec{E}$$

$$\vec{a} = \frac{\vec{F}}{m} = \frac{q\vec{E}}{m}$$

$$\frac{d\vec{v}}{dt} = \frac{qE_0 \sin\omega t}{m} \hat{i}$$

$$\int_0^{\vec{v}} d\vec{v} = \frac{qE_0}{m} \int_0^t \sin\omega t \ \hat{i}$$

$$\vec{v} - 0 = -\frac{qE_0}{m\omega} \left(\cos\omega t - 1\right) \hat{i}$$

$$\boxed{\vec{v} = \frac{2qE_0}{m\omega} \sin^2\frac{\omega t}{2}}$$

$v_{max} \Rightarrow$ sin should be max

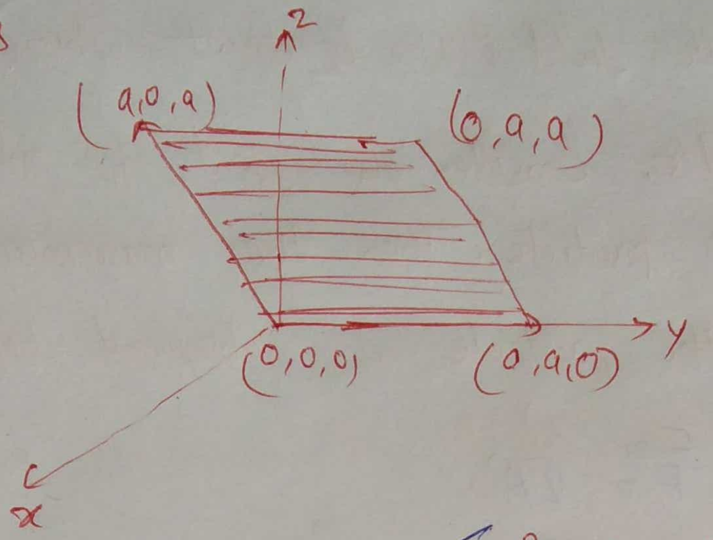
$$\sin^2\frac{\omega t}{2} \approx 1$$

$$\therefore \boxed{v_{max} = \frac{2qE_0}{m\omega}}$$

putting values, we get

$$v_{max} = \frac{2 \times 1 \times 1.0}{10^{-3} \times 10^3} = 2 \text{ m/s} \quad \text{Ans}$$

5) Consider an electric field $\vec{E} = E_0\hat{i}$ where E_0 is a constant. The flux through the shaded region due to this field is



(A) $2E_0a^2$ (B) $\sqrt{2}E_0a^2$ (C) E_0a^2 (D) $\frac{E_0a^2}{2}$

<u>Solution:-</u>

Flux $\phi = \int \vec{E}\cdot d\vec{S} = \vec{E}\cdot\oint d\vec{S}$

$= \vec{E}\cdot\vec{S}$

$\vec{S} = (a\hat{j}) \times (a\hat{i} + a\hat{k})$

$= a^2(\hat{i} - \hat{k})$

$\therefore \phi = E_0\hat{i}\cdot a^2(\hat{i}-\hat{k})$

$$\boxed{\phi = E_0a^2}$$