Show that $\sqrt{3}=1+\frac{1}{3}+\frac{1}{3}\cdot\frac{3}{6}+\frac{1}{3}\cdot\frac{3}{6}\cdot\frac{5}{9}+\frac{1}{3}\cdot\frac{3}{6}\cdot\frac{5}{9}\cdot\frac{7}{12}+\ldots\ldots$

Explanation

Given, $\sqrt{3}$

$\sqrt{3}=(3)^{\frac{1}{2}}=\left(\frac{1}{3}\right)^{-\frac{1}{2}}=\left(1-\frac{2}{3}\right)^{-\frac{1}{2}}$

We know that, If $|x|<1$ & $n\in Q$, then

$(1-x)^n=1+nx+\frac{n(n-1)}{2!}x^2+\frac{n(n-1)(n-2)}{3!}x^3+\frac{n(n-1)(n-2)(n-3)}{4!}x^4+\ldots\ldots,$

$\Rightarrow\left(1-\frac{2}{3}\right)^{-\frac{1}{2}}=1+\left(-\frac{1}{2}\right)\left(-\frac{2}{3}\right)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)}{2!}\left(-\frac{2}{3}\right)^2+\frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)\left(-\frac{1}{2}-2\right)}{3!}\left(-\frac{2}{3}\right)^3$
$+\frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)\left(-\frac{1}{2}-2\right)\left(-\frac{1}{2}-3\right)}{4!}\left(-\frac{2}{3}\right)^4+\ldots\ldots$

$=1+\frac{1}{3}+\left(\frac{1}{2}\right)\left(\frac{3}{4}\right)\left(\frac{1}{2}\right)\left(\frac{4}{9}\right)+\left(\frac{1}{2}\right)\left(\frac{3}{4}\right)\left(\frac{5}{2}\right)\left(\frac{1}{6}\right)\left(\frac{8}{27}\right)+\left(\frac{1}{2}\right)\left(\frac{3}{4}\right)\left(\frac{5}{2}\right)\left(\frac{7}{2}\right)\left(\frac{1}{24}\right)\left(\frac{16}{81}\right)+\ldots\ldots$

$=1+\frac{1}{3}+\frac{1}{3}\cdot\frac{3}{6}+\frac{1}{3}\cdot\frac{3}{6}\cdot\frac{5}{9}+\frac{1}{3}\cdot\frac{3}{6}\cdot\frac{5}{9}\cdot\frac{7}{12}+\ldots\ldots$