

question for practice Find the coefficient of x^n in the expansion of $(1 - 9x + 20x^2)^{-1}$.

Sol. We have,

$$\begin{aligned}(1 - 9x + 20x^2)^{-1} &= [(1 - 5x)(1 - 4x)]^{-1} \\ &= \frac{1}{(1 - 5x)(1 - 4x)} = \frac{5}{1 - 5x} - \frac{4}{1 - 4x} \\ &= 5(1 - 5x)^{-1} - 4(1 - 4x)^{-1} \\ &= 5[1 + 5x + (5x)^2 + \dots + (5x)^n + \dots] - 4[1 + 4x + (4x)^2 \\ &\quad + \dots + (4x)^n + \dots]\end{aligned}$$

Therefore the coefficient of x^n is $5^{n+1} - 4^{n+1}$.