Assuming x to be so small that x^2 and higher powers of x can be neglected, prove that

$$\frac{\left(1+\frac{3}{4}x\right)^{-4}(16-3x)^{1/2}}{(8+x)^{2/3}} = 1 - \frac{305}{96}x$$

Sol. We have,

$$\frac{\left(1+\frac{3}{4}x\right)^{-4}(16-3x)^{1/2}}{(8+x)^{2/3}} = \frac{\left(1+\frac{3}{4}x\right)^{-4}(16)^{1/2}\left(1-\frac{3x}{16}\right)^{1/2}}{8^{2/3}\left(1+\frac{x}{8}\right)^{2/3}}$$

$$= \left(1+\frac{3}{4}x\right)^{-4}\left(1-\frac{3x}{16}\right)^{1/2}\left(1+\frac{x}{8}\right)^{-2/3}$$

$$= \left\{1 + (-4)\left(\frac{3}{4}x\right)\right\}\left\{1 + \frac{1}{2}\left(\frac{-3x}{16}\right)\right\}\left\{1 + \left(-\frac{2}{3}\right)\left(\frac{x}{8}\right)\right\}$$

$$= (1 - 3x)\left(1 - \frac{3}{32}x\right)\left(1 - \frac{x}{12}\right)$$

$$= \left(1 - 3x - \frac{3}{32}x\right)\left(1 - \frac{x}{12}\right) \quad \text{[neglecting } x^2\text{]}$$

$$= \left(1 - \frac{99}{32}x\right)\left(1 - \frac{x}{12}\right) = 1 - \frac{99}{32}x - \frac{x}{12} \quad \text{[neglecting } x^2\text{]}$$

$$= 1 - \frac{305}{96}x$$