Consider the data on x taking the values 0, 2, 4, 8, ..., 2^n with frequencies ${}^nC_0$, ${}^nC_1$, ${}^nC_2$, ..., ${}^nC_n$ respectively. If the mean of this data is $\frac{728}{2^n}$, then n is equal to _____.

[Main Sep. 06, 2020 (II)]

(6.00)

$$\text{Mean} = \frac{\Sigma x_i f_i}{\Sigma f_i} = \frac{0 \cdot {}^nC_0 + 2 \cdot {}^nC_1 + 2^2 \cdot {}^nC_2 + \ldots + 2^n \cdot {}^nC_n}{{}^nC_0 + {}^nC_1 + \ldots + {}^nC_n}$$

To find sum of numerator consider

$$(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \ldots + {}^nC_n x^n \qquad \ldots(i)$$

Put $x = 2 \Rightarrow 3^n - 1 = 2 \cdot {}^nC_1 + 2^2 \cdot {}^nC_2 + \ldots + 2^n \cdot {}^nC_n$

To find sum of denominator, put $x = 1$ in (i), we get

$$2^n = {}^nC_0 + {}^nC_1 + \ldots + {}^nC_n$$

$$\therefore \frac{3^n - 1}{2^n} = \frac{728}{2^n} \Rightarrow 3^n = 729 \Rightarrow n = 6$$