Let $\log_3 2$, $\log_3 2^x - 5$, and $\log_3(2^x - 7/2)$ are in AP

SOLUTION :

Given that $\log_3 2$, $\log_3 (2^x - 5)$, $\log_3 (2^x - 7/2)$ are in A.P. Hence,

$$2 \log_3 (2^x - 5) = \log_3 \left(2^x - \frac{7}{2}\right) + \log_3 2$$

$$\Rightarrow \quad (2^x - 5)^2 = 2\left(2^x - \frac{7}{2}\right)$$

$$\Rightarrow \quad (2^x)^2 - 10 \times 2^x + 25 - 2 \times 2^x + 7 = 0$$

$$\Rightarrow \quad (2^x)^2 - 12 \times 2^x + 32 = 0$$

Let $2^x = y$. Then we get

$$y^2 - 12y + 32 = 0$$

$$\Rightarrow \quad (y - 4)(y - 8) = 0$$

$$\Rightarrow \quad y = 4 \text{ or } 8$$

$$\Rightarrow \quad 2^x = 2^2 \text{ or } 2^3$$

$$\Rightarrow \quad x = 2 \text{ or } 3$$

But for $\log_3(2^x - 5)$ and $\log_3 (2^x - 7/2)$ to be defined,

$$2^x - 5 > 0 \text{ and } 2^x - \frac{7}{2} > 0$$

Therefore x = 3