Let the harmonic mean and geometric mean of two positive numbers be in the ratio 4 : 5 ,then the two numbers are in the ratio **[IIT JEE 1992]**

SOLUTION :

. Let a and b be two positive numbers. Then, H.M. $= \frac{2ab}{a+b}$

and G.M. $= \sqrt{ab}$. According to question, H.M.:G.M. = 4:5

$$\therefore \quad \frac{2ab}{(a+b)\sqrt{ab}} = \frac{4}{5}$$

$$\Rightarrow \quad \frac{2\sqrt{ab}}{a+b} = \frac{4}{5}$$

$$\Rightarrow \quad \frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \frac{5+4}{5-4}$$

$$\Rightarrow \quad \left(\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}}\right)^2 = 9$$

$$\Rightarrow \quad \frac{(\sqrt{a}+\sqrt{b})}{\sqrt{a}-\sqrt{b}} = 3, -3$$

$$\Rightarrow \quad \frac{2\sqrt{a}}{2\sqrt{b}} = \frac{3+1}{3-1}, \frac{-3+1}{-3-1}$$

$$\Rightarrow \quad \frac{\sqrt{a}}{\sqrt{b}} = 2, \frac{1}{2} \Rightarrow \frac{a}{b} = 4, \frac{1}{4}$$

$$\Rightarrow \quad a:b = 4:1 \text{ or } 1:4$$