

If m is AM of two distinct real numbers l and n ($l, m > 1$) and G_1, G_2, G_3 are the three geometric means between l and n then, $G_1^4 + 2G_2^4 + G_3^4$ equals : [JEE MAINS 2015]

- a) $4l^2m^2n^2$ b) $4l^2mn$ c) $4lm^2n$ d) $4lmn^2$

SOLUTION :

l, G_1, G_2, G_3, n are in GP

$$\Rightarrow n = l(r)^4$$

$$\Rightarrow r^4 = \frac{n}{l}$$

$$G_1^4 = (lr)^4 = l^4 r^4 = nl^3$$

$$G_2^4 = (lr^2)^4 = l^4 r^8 = n^2 l^2$$

$$G_3^4 = (lr^3)^4 = l^4 r^{12} = nl^3$$

m is AM of l and n

$$\Rightarrow 2m = l + n \quad \text{--- (1)}$$

now,

$$G_1^4 + 2G_2^4 + G_3^4$$

$$= nl^3 + 2n^2l^2 + nl^3$$

$$= nl(l^2 + 2nl + n^2)$$

$$= nl(l+n)^2$$

$$= nl(2m)^2 \quad \text{(from (1))}$$

$$= 4nlm^2$$