Example 10.7 **Blood velocity:** The flow of blood in a large artery of an anesthetised dog is diverted through a Venturi meter. The wider part of the meter has a cross-sectional area equal to that of the artery. A = 8 mm². The narrower part has an area a = 4 mm². The pressure drop in the artery is 24 Pa. What is the speed of the blood in the artery?

Answer We take the density of blood from Table 10.1 to be 1.06×10^3 kg m^{-3}. The ratio of the areas is $\left(\dfrac{A}{a}\right) = 2$. Using Eq. (10.17) we obtain

$$v_1 = \sqrt{\frac{2 \times 24\,\text{Pa}}{1060\,\text{kg m}^{-3} \times (2^2 - 1)}} = 0.123\,\text{m s}^{-1}$$

10.4.3 Blood Flow and Heart Attack

Bernoulli's principle helps in explaining blood flow in artery. The artery may get constricted due to the accumulation of plaque on its inner walls. In order to drive the blood through this constriction a greater demand is placed on the activity of the heart. The speed of the flow of the blood in this region is raised which lowers the pressure inside and the artery may collapse due to the external pressure. The heart exerts further pressure to open this artery and forces the blood through. As the blood rushes through the opening, the internal pressure once again drops due to same reasons leading to a repeat collapse. This may result in heart attack.

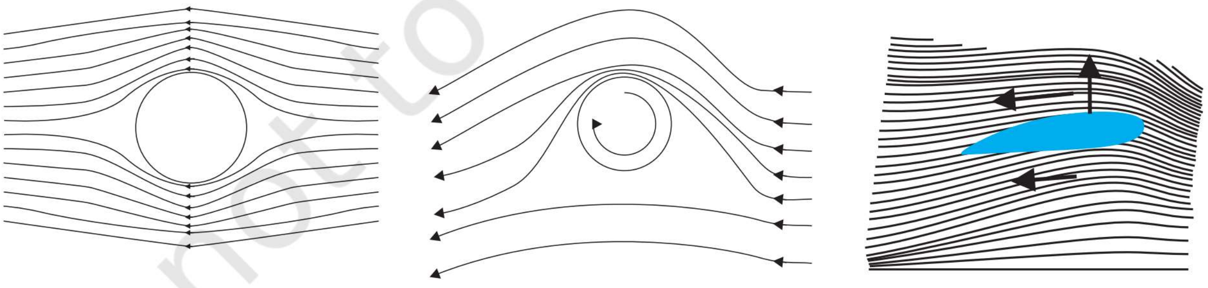
10.4.4 Dynamic Lift

Dynamic lift is the force that acts on a body, such as airplane wing, a hydrofoil or a spinning ball, by virtue of its motion through a fluid. In many games such as cricket, tennis, baseball, or golf, we notice that a spinning ball deviates from its parabolic trajectory as it moves through air. This deviation can be partly explained on the basis of Bernoulli's principle.

(i) **Ball moving without spin**: Fig. 10.13(a) shows the streamlines around a non-spinning ball moving relative to a fluid. From the symmetry of streamlines it is clear that the velocity of fluid (air) above and below the ball at corresponding points is the same resulting in zero pressure difference. The air therefore, exerts no upward or downward force on the ball.

(ii) **Ball moving with spin**: A ball which is spinning drags air along with it. If the surface is rough more air will be dragged. Fig 10.13(b) shows the streamlines of air for a ball which is moving and spinning at the same time. The ball is moving forward and relative to it the air is moving backwards. Therefore, the velocity of air above the ball relative to the ball is larger and below it is smaller (see Section 10.3). The stream lines, thus, get crowded above and rarified below.

This difference in the velocities of air results in the pressure difference between the lower and upper faces and there is a net upward force on the ball. This dynamic lift due to spining is called **Magnus effect**.



(a) (b) (c)

Fig 10.13 *(a) Fluid streaming past a static sphere. (b) Streamlines for a fluid around a sphere spinning clockwise. (c) Air flowing past an aerofoil.*

Aerofoil or lift on aircraft wing: Figure 10.13 (c) shows an aerofoil, which is a solid piece shaped to provide an upward dynamic lift when it moves horizontally through air. The cross-section of the wings of an aeroplane looks somewhat like the aerofoil shown in Fig. 10.13 (c) with streamlines around it. When the aerofoil moves against the wind, the orientation of the wing relative to flow direction causes the streamlines to crowd together above the wing more than those below it. The flow speed on top is higher than that below it. There is an upward force resulting in a dynamic lift of the wings and this balances the weight of the plane. The following example illustrates this.

Example 10.8 A fully loaded Boeing aircraft has a mass of 3.3×10^5 kg. Its total wing area is 500 m^2. It is in level flight with a speed of 960 km/h. (a) Estimate the pressure difference between the lower and upper surfaces of the wings (b) Estimate the fractional increase in the speed of the air on the upper surface of the wing relative to the lower surface. [The density of air is $\rho = 1.2$ kg m^{-3}]

Answer (a) The weight of the Boeing aircraft is balanced by the upward force due to the pressure difference

$$\Delta P \times A = 3.3 \times 10^5 \text{kg} \times 9.8$$

$$\Delta P = (3.3 \times 10^5 \text{ kg} \times 9.8 \text{ m s}^{-2}) / 500 \text{ m}^2$$

$$= 6.5 \times 10^3 \text{ Nm}^{-2}$$

(b) We ignore the small height difference between the top and bottom sides in Eq. (10.12). The pressure difference between them is then

$$\Delta P = \frac{\rho}{2}\left(v_2^2 - v_1^2\right)$$

where v_2 is the speed of air over the upper surface and v_1 is the speed under the bottom surface.

$$\left(v_2 - v_1\right) = \frac{2\Delta P}{\rho\left(v_2 + v_1\right)}$$

Taking the average speed

$v_{av} = (v_2 + v_1)/2 = 960$ km/h $= 267$ m s^{-1},

we have

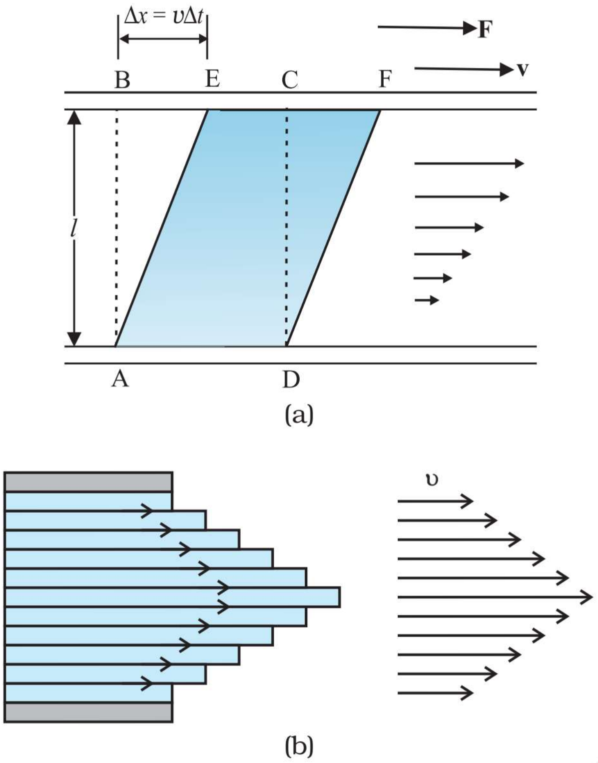
$$\left(v_2 - v_1\right) / v_{av} = \frac{\Delta P}{\rho v_{av}^2} \approx 0.08$$

The speed above the wing needs to be only 8 % higher than that below. ◀

10.5 VISCOSITY

Most of the fluids are not ideal ones and offer some resistance to motion. This resistance to fluid motion is like an internal friction analogous to friction when a solid moves on a surface. It is called viscosity. This force exists when there is relative motion between layers of the liquid. Suppose we consider a fluid like oil enclosed between two glass plates as shown in Fig. 10.14 (a). The bottom plate is fixed while the top plate is moved with a constant velocity **v** relative to the fixed plate. If oil is replaced by honey, a greater force is required to move the plate with the same velocity. Hence we say that honey is more viscous than oil. The fluid in contact with a surface has the same velocity as that of the surfaces. Hence, the layer of the liquid in contact with top surface moves with a velocity **v** and the layer of the liquid in contact with the fixed surface is stationary. The velocities of layers increase uniformly from bottom (zero velocity) to the top layer (velocity **v**). For any layer of liquid, its upper layer pulls it forward while lower layer pulls it backward. This results in force between the layers. This type of flow is known as laminar. The layers of liquid slide over one another as the pages of a book do when it is placed flat on a table and a horizontal force is applied to the top cover. When a fluid is flowing in a pipe or a tube, then velocity of the liquid layer along the axis of the tube is maximum and decreases gradually as we move towards the walls where it becomes zero, Fig. 10.14 (b). The velocity on a cylindrical surface in a tube is constant.

On account of this motion, a portion of liquid, which at some instant has the shape ABCD, take the shape of AEFD after short interval of time



(a)

(b)

Fig 10.14 *(a) A layer of liquid sandwiched between two parallel glass plates, in which the lower plate is fixed and the upper one is moving to the right with velocity* **v**
(b) velocity distribution for viscous flow in a pipe.

(Δt). During this time interval the liquid has undergone a shear strain of $\Delta x/l$. Since, the strain in a flowing fluid increases with time continuously. Unlike a solid, here the stress is found experimentally to depend on 'rate of change of strain' or 'strain rate' i.e. $\Delta x/(l\ \Delta t)$ or v/l instead of strain itself. The coefficient of viscosity (pronounced 'eta') for a fluid is defined as the ratio of shearing stress to the strain rate.

$$\eta = \frac{F/A}{v/l} = \frac{Fl}{vA} \qquad (10.18)$$

The SI unit of viscosity is poiseiulle (Pl). Its other units are N s m^{-2} or Pa s. The dimensions of viscosity are [ML^{-1}T^{-1}]. Generally, thin liquids, like water, alcohol, etc., are less viscous than thick liquids, like coal tar, blood, glycerine, etc. The coefficients of viscosity for some common fluids are listed in Table 10.2. We point out two facts about blood and water that you may find interesting. As Table 10.2 indicates, blood is 'thicker' (more viscous) than water. Further, the relative viscosity (η/η_{water}) of blood remains constant between 0 °C and 37 °C.

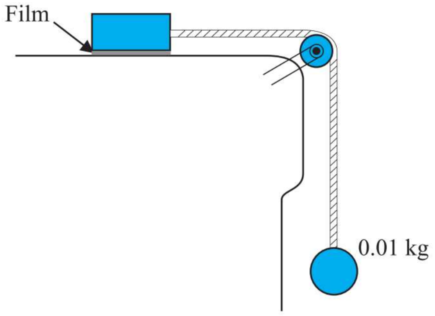


Fig. 10.15 *Measurement of the coefficient of viscosity of a liquid.*

The viscosity of liquids decreases with temperature, while it increases in the case of gases.

Example 10.9 A metal block of area 0.10 m^2 is connected to a 0.010 kg mass via a string that passes over an ideal pulley (considered massless and frictionless), as in Fig. 10.15. A liquid with a film thickness of 0.30 mm is placed between the block and the table. When released the block moves to the right with a constant speed of 0.085 m s^{-1}. Find the coefficient of viscosity of the liquid.

Answer The metal block moves to the right because of the tension in the string. The tension T is equal in magnitude to the weight of the suspended mass m. Thus, the shear force F is

$$F = T = mg = 0.010 \text{ kg} \times 9.8 \text{ m s}^{-2} = 9.8 \times 10^{-2} \text{ N}$$

$$\text{Shear stress on the fluid} = F/A = \frac{9.8 \times 10^{-2}}{0.10} \text{N/m}^2$$

$$\text{Strain rate} = \frac{v}{l} = \frac{0.085}{0.30 \times 10^{-3}}$$

$$\eta = \frac{\text{stress}}{\text{strain rate}} \text{ s}^{-1}$$

$$= \frac{(9.8 \times 10^{-2} \text{ N})\,(0.30 \times 10^{-3} \text{ m})}{(0.085 \text{ m s}^{-1})\,(0.10 \text{ m}^2)}$$

$$= 3.46 \times 10^{-3} \text{ Pa s}$$