Solution Each branch of the network is assigned an unknown current to be determined by the application of Kirchhoff's rules. To reduce the number of unknowns at the outset, the first rule of Kirchhoff is used at every junction to assign the unknown current in each branch. We then have three unknowns I_1, I_2 and I_3 which can be found by applying the second rule of Kirchhoff to three different closed loops. Kirchhoff's second rule for the closed loop ADCA gives,

$$10 - 4(I_1 - I_2) + 2(I_2 + I_3 - I_1) - I_1 = 0 \qquad [3.80(a)]$$

that is, $7I_1 - 6I_2 - 2I_3 = 10$

For the closed loop ABCA, we get

$$10 - 4I_2 - 2\,(I_2 + I_3) - I_1 = 0$$

that is, $I_1 + 6I_2 + 2I_3 = 10$ [3.80(b)]

For the closed loop BCDEB, we get

$$5 - 2\,(I_2 + I_3) - 2\,(I_2 + I_3 - I_1) = 0$$

that is, $2I_1 - 4I_2 - 4I_3 = -5$ [3.80(c)]

Equations (3.80 a, b, c) are three simultaneous equations in three unknowns. These can be solved by the usual method to give

$$I_1 = 2.5\text{A}, \quad I_2 = \frac{5}{8}\text{ A}, \quad I_3 = 1\frac{7}{8}\text{ A}$$

The currents in the various branches of the network are

$$\text{AB}: \frac{5}{8}\text{ A}, \quad \text{CA}: 2\frac{1}{2}\text{ A}, \quad \text{DEB}: 1\frac{7}{8}\text{ A}$$

$$\text{AD}: 1\frac{7}{8}\text{ A}, \quad \text{CD}: 0\text{ A}, \quad \text{BC}: 2\frac{1}{2}\text{ A}$$

It is easily verified that Kirchhoff's second rule applied to the remaining closed loops does not provide any additional independent equation, that is, the above values of currents satisfy the second rule for every closed loop of the network. For example, the total voltage drop over the closed loop BADEB

$$5\text{ V} + \left(\frac{5}{8} \times 4\right)\text{V} - \left(\frac{15}{8} \times 4\right)\text{V}$$

equal to zero, as required by Kirchhoff's second rule.

EXAMPLE 3.7

3.14 Wheatstone Bridge

As an application of Kirchhoff's rules consider the circuit shown in Fig. 3.25, which is called the *Wheatstone bridge*. The bridge has four resistors R_1, R_2, R_3 and R_4. Across one pair of diagonally opposite points (A and C in the figure) a source is connected. This (*i.e.*, AC) is called the battery arm. Between the other two vertices, B and D, a galvanometer G (which is a device to detect currents) is connected. This line, shown as BD in the figure, is called the galvanometer arm.

For simplicity, we assume that the cell has no internal resistance. In general there will be currents flowing across all the resistors as well as a current I_g through G. Of special interest, is the case of a *balanced* bridge where the resistors are such that $I_g = 0$. We can easily get the balance condition, such that there is no current through G. In this case, the Kirchhoff's junction rule applied to junctions D and B (see the figure) immediately gives us the relations $I_1 = I_3$ and $I_2 = I_4$. Next, we apply Kirchhoff's loop rule to closed loops ADBA and CBDC. The first loop gives

$$-I_1 R_1 + 0 + I_2 R_2 = 0 \qquad (I_g = 0) \tag{3.81}$$

and the second loop gives, upon using $I_3 = I_1$, $I_4 = I_2$

$$I_2 R_4 + 0 - I_1 R_3 = 0 \tag{3.82}$$

From Eq. (3.81), we obtain,

$$\frac{I_1}{I_2} = \frac{R_2}{R_1}$$

whereas from Eq. (3.82), we obtain,

$$\frac{I_1}{I_2} = \frac{R_4}{R_3}$$

Hence, we obtain the condition

$$\frac{R_2}{R_1} = \frac{R_4}{R_3} \qquad [3.83(a)]$$

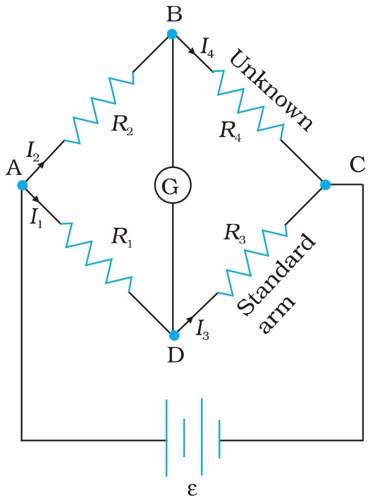

FIGURE 3.25

This last equation relating the four resistors is called the *balance condition* for the galvanometer to give zero or null deflection.

The Wheatstone bridge and its balance condition provide a practical method for determination of an unknown resistance. Let us suppose we have an unknown resistance, which we insert in the fourth arm; R_4 is thus not known. Keeping known resistances R_1 and R_2 in the first and second arm of the bridge, we go on varying R_3 till the galvanometer shows a null deflection. The bridge then is balanced, and from the balance condition the value of the unknown resistance R_4 is given by,

$$R_4 = R_3 \frac{R_2}{R_1} \qquad [3.83(b)]$$

A practical device using this principle is called the *meter bridge*. It will be discussed in the next section.

Example 3.8 The four arms of a Wheatstone bridge (Fig. 3.26) have the following resistances:

AB = 100Ω, BC = 10Ω, CD = 5Ω, and DA = 60Ω.

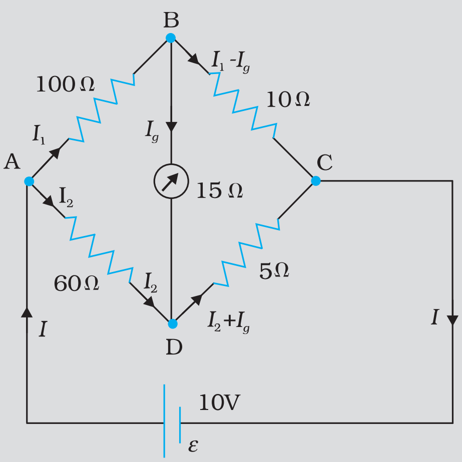

FIGURE 3.26

EXAMPLE 3.8

A galvanometer of 15Ω resistance is connected across BD. Calculate the current through the galvanometer when a potential difference of 10 V is maintained across AC.

Solution Considering the mesh BADB, we have

$$100I_1 + 15I_g - 60I_2 = 0$$

or $$20I_1 + 3I_g - 12I_2 = 0 \qquad [3.84(a)]$$

Considering the mesh BCDB, we have

$$10\,(I_1 - I_g) - 15I_g - 5\,(I_2 + I_g) = 0$$

$$10I_1 - 30I_g - 5I_2 = 0$$

$$2I_1 - 6I_g - I_2 = 0 \qquad [3.84(b)]$$

Considering the mesh ADCEA,

$$60I_2 + 5\,(I_2 + I_g) = 10$$

$$65I_2 + 5I_g = 10$$

$$13I_2 + I_g = 2 \qquad [3.84(c)]$$

Multiplying Eq. (3.84b) by 10

$$20I_1 - 60I_g - 10I_2 = 0 \qquad [3.84(d)]$$

From Eqs. (3.84d) and (3.84a) we have

$$63I_g - 2I_2 = 0$$

$$I_2 = 31.5I_g \qquad [3.84(e)]$$

Substituting the value of I_2 into Eq. [3.84(c)], we get

$$13\,(31.5I_g) + I_g = 2$$

$$410.5\,I_g = 2$$

$$I_g = 4.87 \text{ mA.}$$

EXAMPLE 3.8

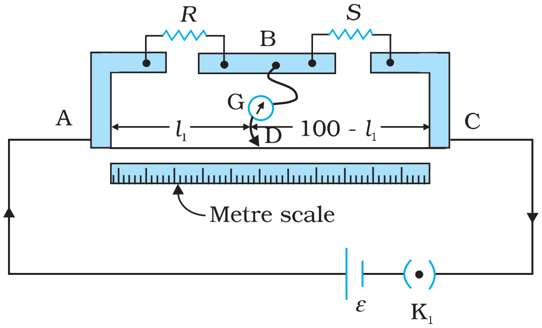

FIGURE 3.27 A meter bridge. Wire AC is 1 m long. *R* is a resistance to be measured and *S* is a standard resistance.

3.15 METER BRIDGE

The meter bridge is shown in Fig. 3.27. It consists of a wire of length 1 m and of uniform cross sectional area stretched taut and clamped between two thick metallic strips bent at right angles, as shown. The metallic strip has two gaps across which resistors can be connected. The end points where the wire is clamped are connected to a cell through a key. One end of a galvanometer is connected to the metallic strip midway between the two gaps. The other end of the galvanometer is connected to a 'jockey'. The jockey is essentially a metallic rod whose one end has a knife-edge which can slide over the wire to make electrical connection.

R is an unknown resistance whose value we want to determine. It is connected across one of the gaps. Across the other gap, we connect a standard known resistance S. The jockey is connected to some point D on the wire, a distance l cm from the end A. The jockey can be moved along the wire. The portion AD of the wire has a resistance $R_{cm}l$, where R_{cm} is the resistance of the wire per unit centimetre. The portion DC of the wire similarly has a resistance R_{cm} (100-l).

The four arms AB, BC, DA and CD [with resistances R, S, $R_{cm}\,l$ and R_{cm}(100-l)] obviously form a Wheatstone bridge with AC as the battery arm and BD the galvanometer arm. If the jockey is moved along the wire, then there will be one position where the galvanometer will show no current. Let the distance of the jockey from the end A at the balance point be $l = l_1$. The four resistances of the bridge at the balance point then are R, S, $R_{cm}\,l_1$ and $R_{cm}(100–l_1)$. The balance condition, Eq. [3.83(a)] gives

$$\frac{R}{S} = \frac{R_{cm}\,l_1}{R_{cm}\,(100 - l_1)} = \frac{l_1}{100 - l_1} \tag{3.85}$$

Thus, once we have found out l_1, the unknown resistance R is known in terms of the standard known resistance S by

$$R = S\,\frac{l_1}{100 - l_1} \tag{3.86}$$

By choosing various values of S, we would get various values of l_1, and calculate R each time. An error in measurement of l_1 would naturally result in an error in R. It can be shown that the percentage error in R can be minimised by adjusting the balance point near the middle of the bridge, i.e., when l_1 is close to 50 cm. (This requires a suitable choice of S.)

Example 3.9 In a meter bridge (Fig. 3.27), the null point is found at a distance of 33.7 cm from A. If now a resistance of 12Ω is connected in parallel with S, the null point occurs at 51.9 cm. Determine the values of R and S.

Solution From the first balance point, we get

$$\frac{R}{S} = \frac{33.7}{66.3} \tag{3.87}$$

After S is connected in parallel with a resistance of 12Ω , the resistance across the gap changes from S to S_{eq}, where

$$S_{eq} = \frac{12S}{S+12}$$

and hence the new balance condition now gives

$$\frac{51.9}{48.1} = \frac{R}{S_{eq}} = \frac{R(S+12)}{12S} \tag{3.88}$$

Substituting the value of R/S from Eq. (3.87), we get

$$\frac{51.9}{48.1} = \frac{S+12}{12} \cdot \frac{33.7}{66.3}$$

which gives S = 13.5Ω. Using the value of R/S above, we get R = 6.86 Ω.

EXAMPLE 3.9

3.16 Potentiometer

This is a versatile instrument. It is basically a long piece of uniform wire, sometimes a few meters in length across which a standard cell (B) is connected. In actual design, the wire is sometimes cut in several pieces placed side by side and connected at the ends by thick metal strip. (Fig. 3.28). In the figure, the wires run from A to C. The small vertical portions are the thick metal strips connecting the various sections of the wire.

A current *I* flows through the wire which can be varied by a variable resistance (rheostat, *R*) in the circuit. Since the wire is uniform, the potential difference between A and any point at a distance *l* from A is

$$\varepsilon(l)=\phi l \tag{3.89}$$

where ϕ is the potential drop per unit length.

Figure 3.28 (a) shows an application of the potentiometer to compare the emf of two cells of emf ε_1 and ε_2 . The points marked 1, 2, 3 form a two way key. Consider first a position of the key where 1 and 3 are connected so that the galvanometer is connected to ε_1. The jockey is moved along the wire till at a point N_1, at a distance l_1 from A, there is no deflection in the galvanometer. We can apply Kirchhoff's loop rule to the closed loop AN_1G31A and get,

$$\phi\, l_1 + 0 - \varepsilon_1 = 0 \tag{3.90}$$

Similarly, if another emf ε_2 is balanced against l_2 (AN_2)

$$\phi\, l_2 + 0 - \varepsilon_2 = 0 \tag{3.91}$$

From the last two equations

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{l_1}{l_2} \tag{3.92}$$

This simple mechanism thus allows one to compare the emf's of any two sources (ε_1,ε_2). In practice one of the cells is chosen as a standard cell whose emf is known to a high degree of accuracy. The emf of the other cell is then easily calculated from Eq. (3.92).

We can also use a potentiometer to measure internal resistance of a cell [Fig. 3.28 (b)]. For this the cell (emf ε) whose internal resistance (*r*) is to be determined is connected across a resistance box through a key K_2, as shown in the figure. With key K_2 open, balance is obtained at length l_1 (AN_1). Then,

$$\varepsilon = \phi\, l_1 \tag{3.93(a)}$$

When key K_2 is closed, the cell sends a current (*I*) through the resistance box (*R*). If *V* is the terminal potential difference of the cell and balance is obtained at length l_2 (AN_2),

$$V = \phi\, l_2 \tag{3.93(b)}$$

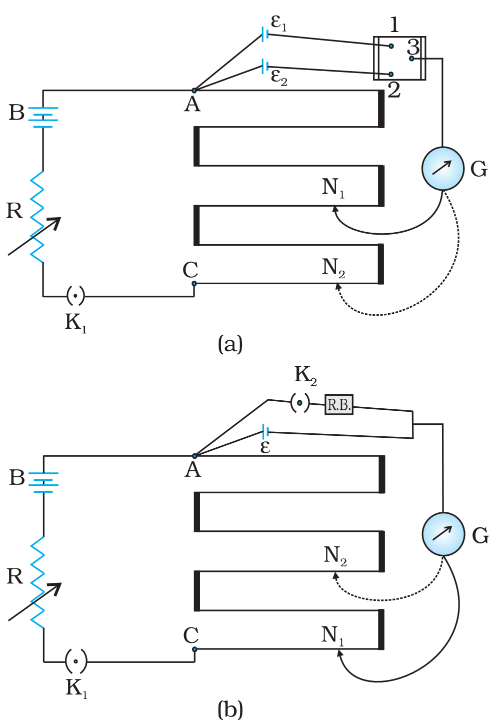

FIGURE 3.28 A potentiometer. *G* is a galvanometer and *R* a variable resistance (rheostat). 1, 2, 3 are terminals of a two way key (a) circuit for comparing emfs of two cells; (b) circuit for determining internal resistance of a cell.

So, we have $\varepsilon/V = l_1/l_2$ [3.94(a)]

But, $\varepsilon = I\,(r + R)$ and $V = IR$. This gives

$\varepsilon/V = (r+R)/R$ [3.94(b)]

From Eq. [3.94(a)] and [3.94(b)] we have

$(R+r)/R = l_1/l_2$

$$r = R\left(\frac{l_1}{l_2} - 1\right) \quad (3.95)$$

Using Eq. (3.95) we can find the internal resistance of a given cell.

The potentiometer has the advantage that it draws *no current* from the voltage source being measured. As such it is unaffected by the internal resistance of the source.

Example 3.10 A resistance of R Ω draws current from a potentiometer. The potentiometer has a total resistance R_0 Ω (Fig. 3.29). A voltage V is supplied to the potentiometer. Derive an expression for the voltage across R when the sliding contact is in the middle of the potentiometer.

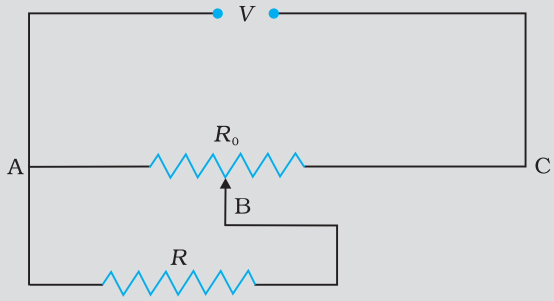

FIGURE 3.29

Solution While the slide is in the middle of the potentiometer only half of its resistance (R_0/2) will be between the points A and B. Hence, the total resistance between A and B, say, R_1, will be given by the following expression:

$$\frac{1}{R_1} = \frac{1}{R} + \frac{1}{(R_0/2)}$$

$$R_1 = \frac{R_0 R}{R_0 + 2R}$$

The total resistance between A and C will be sum of resistance between A and B and B and C, i.e., $R_1 + R_0/2$

∴ The current flowing through the potentiometer will be

$$I = \frac{V}{R_1 + R_0/2} = \frac{2V}{2R_1 + R_0}$$

The voltage V_1 taken from the potentiometer will be the product of current I and resistance R_1,

$$V_1 = I\,R_1 = \left(\frac{2V}{2R_1 + R_0}\right) \times R_1$$

EXAMPLE 3.10

EXAMPLE 3.10

Substituting for R_1, we have a

$$V_1 = \frac{2V}{2\left(\frac{R_0 \times R}{R_0 + 2R}\right) + R_0} \times \frac{R_0 \times R}{R_0 + 2R}$$

$$V_1 = \frac{2VR}{2R + R_0 + 2R}$$

or $$V_1 = \frac{2VR}{R_0 + 4R}.$$

SUMMARY

1. *Current* through a given area of a conductor is the net charge passing per unit time through the area.
2. To maintain a steady current, we must have a closed circuit in which an external agency moves electric charge from lower to higher potential energy. The work done per unit charge by the source in taking the charge from lower to higher potential energy (i.e., from one terminal of the source to the other) is called the electromotive force, or *emf*, of the source. Note that the emf is not a force; it is the voltage difference between the two terminals of a source in open circuit.
3. *Ohm's law*: The electric current I flowing through a substance is proportional to the voltage V across its ends, i.e., $V \propto I$ or $V = RI$, where R is called the *resistance* of the substance. The unit of resistance is ohm: $1\Omega = 1 \text{ V A}^{-1}$.
4. The *resistance* R of a conductor depends on its length l and cross-sectional area A through the relation,

   $$R = \frac{\rho l}{A}$$

   where ρ, called *resistivity* is a property of the material and depends on temperature and pressure.
5. *Electrical resistivity* of substances varies over a very wide range. Metals have low resistivity, in the range of $10^{-8}\ \Omega$ m to $10^{-6}\ \Omega$ m. Insulators like glass and rubber have 10^{22} to 10^{24} times greater resistivity. Semiconductors like Si and Ge lie roughly in the middle range of resistivity on a logarithmic scale.
6. In most substances, the carriers of current are electrons; in some cases, for example, ionic crystals and electrolytic liquids, positive and negative ions carry the electric current.
7. *Current density* **j** gives the amount of charge flowing per second per unit area normal to the flow,

   $$\mathbf{j} = nq\,\mathbf{v}_d$$

   where n is the number density (number per unit volume) of charge carriers each of charge q, and $\mathbf{v}_d$ is the *drift velocity* of the charge carriers. For electrons $q = -e$. If **j** is normal to a cross-sectional area **A** and is constant over the area, the magnitude of the current I through the area is $nev_d A$.

8. Using $E = V/l$, $I = nev_d A$, and Ohm's law, one obtains

$$\frac{eE}{m} = \rho \frac{ne^2}{m} v_d$$

The proportionality between the *force eE* on the electrons in a metal due to the external field E and the drift velocity v_d (not acceleration) can be understood, if we assume that the electrons suffer collisions with ions in the metal, which deflect them randomly. If such collisions occur on an average at a time interval τ,

$$v_d = a\tau = eE\tau/m$$

where a is the acceleration of the electron. This gives

$$\rho = \frac{m}{ne^2 \tau}$$

9. In the temperature range in which resistivity increases linearly with temperature, the *temperature coefficient of resistivity* α is defined as the fractional increase in resistivity per unit increase in temperature.
10. Ohm's law is obeyed by many substances, but it is not a fundamental law of nature. It fails if
    (a) V depends on I non-linearly.
    (b) the relation between V and I depends on the sign of V for the same absolute value of V.
    (c) The relation between V and I is non-unique.

    An example of (a) is when ρ increases with I (even if temperature is kept fixed). A rectifier combines features (a) and (b). GaAs shows the feature (c).
11. When a source of emf ε is connected to an external resistance R, the voltage V_{ext} across R is given by

$$V_{ext} = IR = \frac{\varepsilon}{R + r} R$$

where r is the *internal resistance* of the source.
12. (a) Total resistance R of n resistors connected in *series* is given by

    $$R = R_1 + R_2 + ... + R_n$$

    (b) Total resistance R of n resistors connected in *parallel* is given by

    $$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + ... + \frac{1}{R_n}$$
13. *Kirchhoff's Rules* –
    (a) *Junction Rule:* At any junction of circuit elements, the sum of currents entering the junction must equal the sum of currents leaving it.
    (b) *Loop Rule:* The algebraic sum of changes in potential around any closed loop must be zero.
14. The *Wheatstone bridge* is an arrangement of four resistances – R_1, R_2, R_3, R_4 as shown in the text. The null-point condition is given by

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

using which the value of one resistance can be determined, knowing the other three resistances.
15. The *potentiometer* is a device to compare potential differences. Since the method involves a condition of *no* current flow, the device can be used to measure potential difference; internal resistance of a cell and compare emf's of two sources.

Physical Quantity	Symbol	Dimensions	Unit	Remark
Electric current	I	[A]	A	SI base unit
Charge	Q, q	[T A]	C	
Voltage, Electric potential difference	V	$[M L^2 T^{-3} A^{-1}]$	V	Work/charge
Electromotive force	ε	$[M L^2 T^{-3} A^{-1}]$	V	Work/charge
Resistance	R	$[M L^2 T^{-3} A^{-2}]$	Ω	$R = V/I$
Resistivity	ρ	$[M L^3 T^{-3} A^{-2}]$	Ω m	$R = \rho l/A$
Electrical conductivity	σ	$[M^{-1} L^{-3} T^3 A^2]$	S	$\sigma = 1/\rho$
Electric field	**E**	$[M L T^{-3} A^{-1}]$	$V m^{-1}$	$\frac{\text{Electric force}}{\text{charge}}$
Drift speed	v_d	$[L T^{-1}]$	$m s^{-1}$	$v_d = \frac{e E \tau}{m}$
Relaxation time	τ	[T]	s	
Current density	**j**	$[L^{-2} A]$	$A m^{-2}$	current/area
Mobility	μ	$[M L^3 T^{-4} A^{-1}]$	$m^2 V^{-1} s^{-1}$	v_d / E

POINTS TO PONDER

1. Current is a scalar although we represent current with an arrow. Currents do not obey the law of vector addition. That current is a scalar also follows from it's definition. The current I through an area of cross-section is given by the scalar product of two vectors:

   $I = \mathbf{j} \cdot \Delta\mathbf{S}$

   where **j** and Δ**S** are vectors.
2. Refer to *V-I* curves of a resistor and a diode as drawn in the text. A resistor obeys Ohm's law while a diode does not. The assertion that $V = IR$ is a statement of Ohm's law is not true. This equation defines resistance and it may be applied to all conducting devices whether they obey Ohm's law or not. The Ohm's law asserts that the plot of I versus V is linear i.e., R is independent of V.

   Equation $\mathbf{E} = \rho\, \mathbf{j}$ leads to another statement of *Ohm's* law, i.e., a conducting material obeys Ohm's law when the resistivity of the material does not depend on the magnitude and direction of applied electric field.
3. Homogeneous conductors like silver or semiconductors like pure germanium or germanium containing impurities obey Ohm's law within some range of electric field values. If the field becomes too strong, there are departures from Ohm's law in all cases.
4. Motion of conduction electrons in electric field **E** is the sum of (i) motion due to random collisions and (ii) that due to **E**. The motion