Answer Figures 5.8(b) and 5.8(c) are known as free-body diagrams. Figure 5.8(b) is the free-body diagram of W and Fig. 5.8(c) is the free-body diagram of point P.

Consider the equilibrium of the weight W. Clearly, $T_2 = 6 \times 10 = 60$ N.

Consider the equilibrium of the point P under the action of three forces - the tensions T_1 and T_2, and the horizontal force 50 N. The horizontal and vertical components of the resultant force must vanish separately :

$$T_1 \cos\theta = T_2 = 60 \text{ N}$$

$$T_1 \sin\theta = 50 \text{ N}$$

which gives that

$$\tan\theta = \frac{5}{6} \text{ or } \theta = \tan^{-1}\left(\frac{5}{6}\right) = 40^{\circ}$$

Note the answer does not depend on the length of the rope (assumed massless) nor on the point at which the horizontal force is applied. ◀

5.9 COMMON FORCES IN MECHANICS

In mechanics, we encounter several kinds of forces. The gravitational force is, of course, pervasive. Every object on the earth experiences the force of gravity due to the earth. Gravity also governs the motion of celestial bodies. The gravitational force can act at a distance without the need of any intervening medium.

All the other forces common in mechanics are contact forces.* As the name suggests, a contact force on an object arises due to contact with some other object: solid or fluid. When bodies are in contact (e.g. a book resting on a table, a system of rigid bodies connected by rods, hinges and other types of supports), there are mutual contact forces (for each pair of bodies) satisfying the third law. The component of contact force normal to the surfaces in contact is called normal reaction. The component parallel to the surfaces in contact is called friction. Contact forces arise also when solids are in contact with fluids. For example, for a solid immersed in a fluid, there is an upward bouyant force equal to the weight of the fluid displaced. The viscous force, air resistance, etc are also examples of contact forces (Fig. 5.9).

Two other common forces are tension in a string and the force due to spring. When a spring is compressed or extended by an external force, a restoring force is generated. This force is usually proportional to the compression or elongation (for small displacements). The spring force F is written as $F = -k\,x$ where x is the displacement and k is the force constant. The negative sign denotes that the force is opposite to the displacement from the unstretched state. For an inextensible string, the force constant is very high. The restoring force in a string is called tension. It is customary to use a constant tension T throughout the string. This assumption is true for a string of negligible mass.

In Chapter 1, we learnt that there are four fundamental forces in nature. Of these, the weak and strong forces appear in domains that do not concern us here. Only the gravitational and electrical forces are relevant in the context of mechanics. The different contact forces of mechanics mentioned above fundamentally arise from electrical forces. This may seem surprising

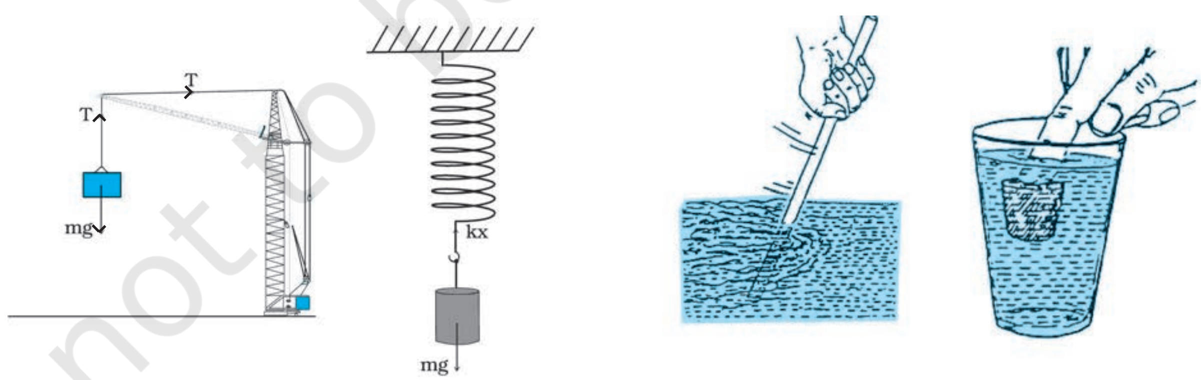

Fig. 5.9 *Some examples of contact forces in mechanics.*

* *We are not considering, for simplicity, charged and magnetic bodies. For these, besides gravity, there are electrical and magnetic non-contact forces.*

since we are talking of uncharged and non-magnetic bodies in mechanics. At the microscopic level, all bodies are made of charged constituents (nuclei and electrons) and the various contact forces arising due to elasticity of bodies, molecular collisions and impacts, etc. can ultimately be traced to the electrical forces between the charged constituents of different bodies. The detailed microscopic origin of these forces is, however, complex and not useful for handling problems in mechanics at the macroscopic scale. This is why they are treated as different types of forces with their characteristic properties determined empirically.

5.9.1 Friction

Let us return to the example of a body of mass m at rest on a horizontal table. The force of gravity (mg) is cancelled by the normal reaction force (N) of the table. Now suppose a force F is applied horizontally to the body. We know from experience that a small applied force may not be enough to move the body. But if the applied force F were the only external force on the body, it must move with acceleration F/m, however small. Clearly, the body remains at rest because some other force comes into play in the horizontal direction and opposes the applied force F, resulting in zero net force on the body. This force f_s parallel to the surface of the body in contact with the table is known as frictional force, or simply friction (Fig. 5.10(a)). The subscript stands for static friction to distinguish it from kinetic friction f_k that we consider later (Fig. 5.10(b)). Note that static friction does not

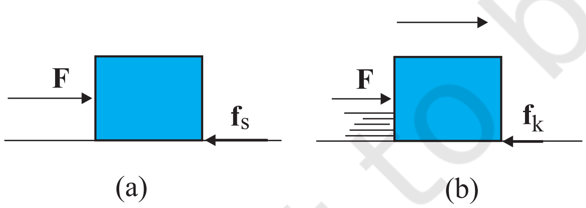

Fig. 5.10 *Static and sliding friction: (a) Impending motion of the body is opposed by static friction. When external force exceeds the maximum limit of static friction, the body begins to move. (b) Once the body is in motion, it is subject to sliding or kinetic friction which opposes relative motion between the two surfaces in contact. Kinetic friction is usually less than the maximum value of static friction.*

exist by itself. When there is no applied force, there is no static friction. It comes into play the moment there is an applied force. As the applied force F increases, f_s also increases, remaining equal and opposite to the applied force (up to a certain limit), keeping the body at rest. Hence, it is called **static friction**. Static friction opposes **impending motion**. The term impending motion means motion that would take place (but does not actually take place) under the applied force, if friction were absent.

We know from experience that as the applied force exceeds a certain limit, the body begins to move. It is found experimentally that the limiting value of static friction $(f_s)_{max}$ is independent of the area of contact and varies with the normal force (N) approximately as :

$$(f_s)_{max} = \mu_s N \qquad (5.13)$$

where μ_s is a constant of proportionality depending only on the nature of the surfaces in contact. The constant μ_s is called the coefficient of static friction. The law of static friction may thus be written as

$$f_s \le \mu_s N \qquad (5.14)$$

If the applied force F exceeds $(f_s)_{max}$ the body begins to slide on the surface. It is found experimentally that when relative motion has started, the frictional force decreases from the static maximum value $(f_s)_{max}$. Frictional force that opposes relative motion between surfaces in contact is called kinetic or sliding friction and is denoted by $\mathbf{f}_k$. Kinetic friction, like static friction, is found to be independent of the area of contact. Further, it is nearly independent of the velocity. It satisfies a law similar to that for static friction:

$$\mathbf{f}_k = \mu_k \mathbf{N} \qquad (5.15)$$

where μ_k, the coefficient of kinetic friction, depends only on the surfaces in contact. As mentioned above, experiments show that μ_k is less than μ_s. When relative motion has begun, the acceleration of the body according to the second law is $(F - f_k)/m$. For a body moving with constant velocity, $F = f_k$. If the applied force on the body is removed, its acceleration is $-f_k/m$ and it eventually comes to a stop.

The laws of friction given above do not have the status of fundamental laws like those for gravitational, electric and magnetic forces. They are empirical relations that are only approximately true. Yet they are very useful in practical calculations in mechanics.

Thus, when two bodies are in contact, each experiences a contact force by the other. Friction, by definition, is the component of the contact force parallel to the surfaces in contact, which opposes impending or actual relative motion between the two surfaces. Note that it is not motion, but **relative motion** that the frictional force opposes. Consider a box lying in the compartment of a train that is accelerating. If the box is stationary relative to the train, it is in fact accelerating along with the train. What forces cause the acceleration of the box? Clearly, the only conceivable force in the horizontal direction is the force of friction. If there were no friction, the floor of the train would slip by and the box would remain at its initial position due to inertia (and hit the back side of the train). This impending relative motion is opposed by the static friction f_s. Static friction provides the same acceleration to the box as that of the train, keeping it stationary relative to the train.

Example 5.7 Determine the maximum acceleration of the train in which a box lying on its floor will remain stationary, given that the co-efficient of static friction between the box and the train's floor is 0.15.

Answer Since the acceleration of the box is due to the static friction,

$$ma = f_s \le \mu_s N = \mu_s \, mg$$

i.e. $a \le \mu_s g$

$$\therefore a_{max} = \mu_s g = 0.15 \times 10 \text{ m s}^{-2} = 1.5 \text{ m s}^{-2}$$

Example 5.8 See Fig. 5.11. A mass of 4 kg rests on a horizontal plane. The plane is gradually inclined until at an angle $\theta = 15°$ with the horizontal, the mass just begins to slide. What is the coefficient of static friction between the block and the surface ?

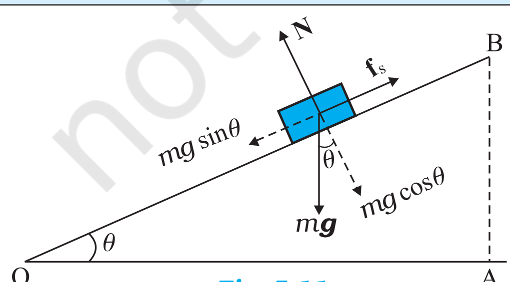

Fig. 5.11

Answer The forces acting on a block of mass *m* at rest on an inclined plane are (i) the weight mg acting vertically downwards (ii) the normal force *N* of the plane on the block, and (iii) the static frictional force f_s opposing the impending motion. In equilibrium, the resultant of these forces must be zero. Resolving the weight *mg* along the two directions shown, we have

$$mg \sin\theta = f_s \ , \quad mg \cos\theta = N$$

As θ increases, the self-adjusting frictional force f_s increases until at $\theta = \theta_{max}$, f_s achieves its maximum value, $(f_s)_{max} = \mu_s N$.

Therefore,

$$\tan\theta_{max} = \mu_s \ \text{ or } \ \theta_{max} = \tan^{-1}\mu_s$$

When θ becomes just a little more than θ_{max}, there is a small net force on the block and it begins to slide. Note that θ_{max} depends only on μ_s and is independent of the mass of the block.

For $\theta_{max} = 15°$,
$\mu_s = \tan 15° = 0.27$

Example 5.9 What is the acceleration of the block and trolley system shown in a Fig. 5.12(a), if the coefficient of kinetic friction between the trolley and the surface is 0.04? What is the tension in the string? (Take g = 10 m s^{-2}). Neglect the mass of the string.

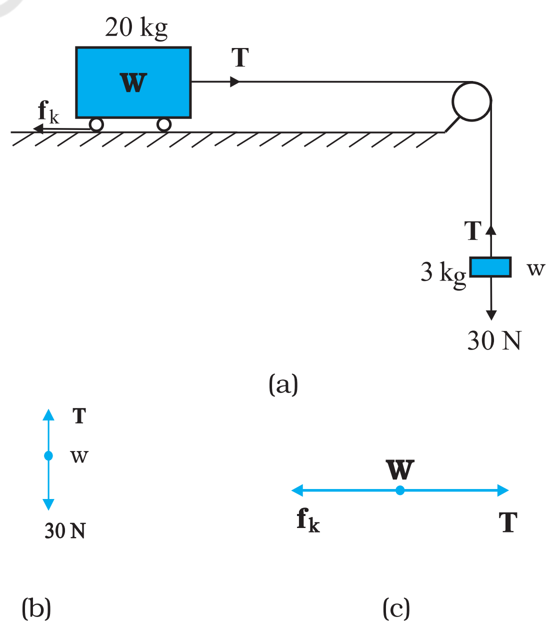

Fig. 5.12

Answer As the string is inextensible, and the pully is smooth, the 3 kg block and the 20 kg trolley both have same magnitude of acceleration. Applying second law to motion of the block (Fig. 5.12(b)),

$$30 - T = 3a$$

Apply the second law to motion of the trolley (Fig. 5.12(c)),

$$T - f_k = 20\ a.$$

Now $f_k = \mu_k N,$

Here $\mu_k = 0.04,$

$N = 20 \times 10$

$= 200$ N.

Thus the equation for the motion of the trolley is

$$T - 0.04 \times 200 = 20\ a \text{ Or } T - 8 = 20a.$$

These equations give $a = \frac{22}{23}$ m s^{-2} = 0.96 m s^{-2}

and T = 27.1 N. ◀

Rolling friction

A body like a ring or a sphere rolling without slipping over a horizontal plane will suffer no friction, in principle. At every instant, there is just one point of contact between the body and the plane and this point has no motion relative to the plane. In this ideal situation, kinetic or static friction is zero and the body should continue to roll with constant velocity. We know, in practice, this will not happen and some resistance to motion (rolling friction) does occur, i.e. to keep the body rolling, some applied force is needed. For the same weight, rolling friction is much smaller (even by 2 or 3 orders of magnitude) than static or sliding friction. This is the reason why discovery of the wheel has been a major milestone in human history.

Rolling friction again has a complex origin, though somewhat different from that of static and sliding friction. During rolling, the surfaces in contact get momentarily deformed a little, and this results in a finite area (not a point) of the body being in contact with the surface. The net effect is that the component of the contact force parallel to the surface opposes motion.

We often regard friction as something undesirable. In many situations, like in a machine with different moving parts, friction does have a negative role. It opposes relative motion and thereby dissipates power in the form of heat, etc. Lubricants are a way of reducing kinetic friction in a machine. Another way is to use ball bearings between two moving parts of a machine [Fig. 5.13(a)]. Since the rolling friction between ball bearings and the surfaces in contact is very small, power dissipation is reduced. A thin cushion of air maintained between solid surfaces in relative motion is another effective way of reducing friction (Fig. 5.13(a)).

In many practical situations, however, friction is critically needed. Kinetic friction that dissipates power is nevertheless important for quickly stopping relative motion. It is made use of by brakes in machines and automobiles. Similarly, static friction is important in daily life. We are able to walk because of friction. It is impossible for a car to move on a very slippery road. On an ordinary road, the friction between the tyres and the road provides the necessary external force to accelerate the car.

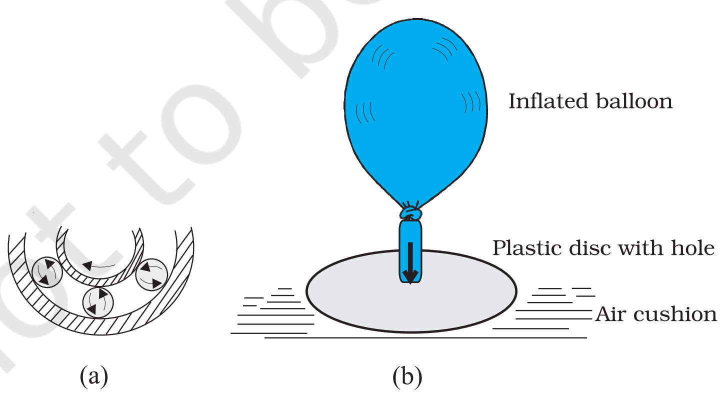

Fig. 5.13 *Some ways of reducing friction. (a) Ball bearings placed between moving parts of a machine. (b) Compressed cushion of air between surfaces in relative motion.*

5.10 CIRCULAR MOTION

We have seen in Chapter 4 that acceleration of a body moving in a circle of radius R with uniform speed v is v^2/R directed towards the centre. According to the second law, the force f_c providing this acceleration is :

$$f_c = \frac{mv^2}{R} \qquad (5.16)$$

where m is the mass of the body. This force directed forwards the centre is called the centripetal force. For a stone rotated in a circle by a string, the centripetal force is provided by the tension in the string. The centripetal force for motion of a planet around the sun is the gravitational force on the planet due to the sun. For a car taking a circular turn on a horizontal road, the centripetal force is the force of friction.

The circular motion of a car on a flat and banked road give interesting application of the laws of motion.

Motion of a car on a level road

Three forces act on the car (Fig. 5.14(a):

(i) The weight of the car, mg

(ii) Normal reaction, N

(iii) Frictional force, f

As there is no acceleration in the vertical direction

$N - mg = 0$

$N = mg \qquad (5.17)$

The centripetal force required for circular motion is along the surface of the road, and is provided by the component of the contact force between road and the car tyres along the surface. This by definition is the frictional force. Note that it is the static friction that provides the centripetal acceleration. Static friction opposes the impending motion of the car moving away from the circle. Using equation (5.14) & (5.16) we get the result

$$f = \frac{mv^2}{R} \le \mu_s N$$

$$v^2 \le \frac{\mu_s RN}{m} = \mu_s Rg \qquad [\because N = mg]$$

which is independent of the mass of the car. This shows that for a given value of μ_s and R, there is a maximum speed of circular motion of the car possible, namely

$$v_{\max} = \sqrt{\mu_s Rg} \qquad (5.18)$$

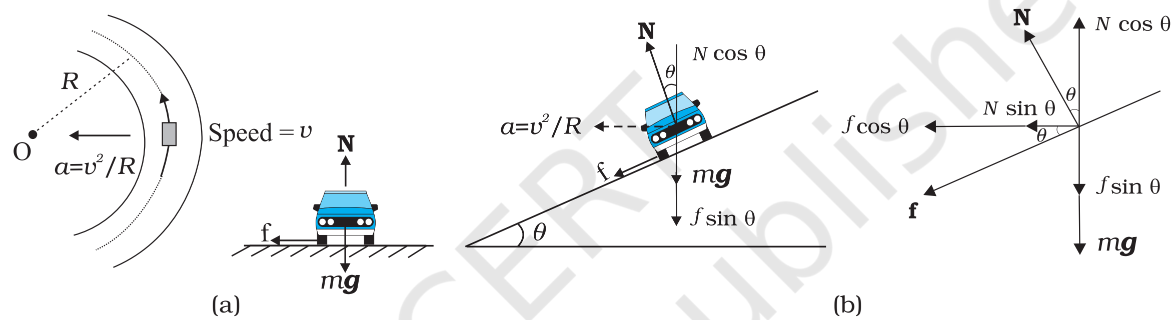

Fig. 5.14 *Circular motion of a car on (a) a level road, (b) a banked road.*

Motion of a car on a banked road

We can reduce the contribution of friction to the circular motion of the car if the road is banked (Fig. 5.14(b)). Since there is no acceleration along the vertical direction, the net force along this direction must be zero. Hence,

$$N\cos\theta = mg + f\sin\theta \qquad (5.19a)$$

The centripetal force is provided by the horizontal components of N and f.

$$N\sin\theta + f\cos\theta = \frac{mv^2}{R} \qquad (5.19b)$$

But $f \le \mu_s N$

Thus to obtain v_{max} we put

$f = \mu_s N$.

Then Eqs. (5.19a) and (5.19b) become

$$N\cos\theta = mg + \mu_s N\sin\theta \qquad (5.20a)$$

$N \sin\theta + \mu_s N \cos\theta = mv^2/R$ (5.20b)

From Eq. (5.20a), we obtain

$$N = \frac{mg}{\cos\theta - \mu_s \sin\theta}$$

Substituting value of N in Eq. (5.20b), we get

$$\frac{mg(\sin\theta + \mu_s \cos\theta)}{\cos\theta - \mu_s \sin\theta} = \frac{mv_{max}^2}{R}$$

$$\text{or } v_{max} = \left(Rg \frac{\mu_s + \tan\theta}{1 - \mu_s \tan\theta}\right)^{\frac{1}{2}} \quad (5.21)$$

Comparing this with Eq. (5.18) we see that maximum possible speed of a car on a banked road is greater than that on a flat road.

For $\mu_s = 0$ in Eq. (5.21),

$$v_o = (R g \tan\theta)^{1/2} \quad (5.22)$$

At this speed, frictional force is not needed at all to provide the necessary centripetal force. Driving at this speed on a banked road will cause little wear and tear of the tyres. The same equation also tells you that for $v < v_o$, frictional force will be up the slope and that a car can be parked only if $\tan\theta \le \mu_s$.

▶ ***Example 5.10*** A cyclist speeding at 18 km/h on a level road takes a sharp circular turn of radius 3 m without reducing the speed. The co-efficient of static friction between the tyres and the road is 0.1. Will the cyclist slip while taking the turn?

Answer On an unbanked road, frictional force alone can provide the centripetal force needed to keep the cyclist moving on a circular turn without slipping. If the speed is too large, or if the turn is too sharp (i.e. of too small a radius) or both, the frictional force is not sufficient to provide the necessary centripetal force, and the cyclist slips. The condition for the cyclist not to slip is given by Eq. (5.18) :

$$v^2 \le \mu_s R g$$

Now, $R = 3$ m, $g = 9.8$ m s^{-2}, $\mu_s = 0.1$. That is, $\mu_s R g = 2.94$ m^2 s^{-2}. $v = 18$ km/h $= 5$ m s^{-1}; i.e., $v^2 = 25$ m^2 s^{-2}. The condition is not obeyed. The cyclist will slip while taking the circular turn. ◀

▶ ***Example 5.11*** A circular racetrack of radius 300 m is banked at an angle of 15°. If the coefficient of friction between the wheels of a race-car and the road is 0.2, what is the (a) optimum speed of the race-car to avoid wear and tear on its tyres, and (b) maximum permissible speed to avoid slipping ?

Answer On a banked road, the horizontal component of the normal force and the frictional force contribute to provide centripetal force to keep the car moving on a circular turn without slipping. At the optimum speed, the normal reaction's component is enough to provide the needed centripetal force, and the frictional force is not needed. The optimum speed v_o is given by Eq. (5.22):

$$v_O = (R g \tan\theta)^{1/2}$$

Here $R = 300$ m, $\theta = 15°$, $g = 9.8$ m s^{-2}; we have

$$v_O = 28.1 \text{ m s}^{-1}.$$

The maximum permissible speed v_{max} is given by Eq. (5.21):

$$v_{max} = \left(R g \frac{\mu_s + \tan\theta}{1 - \mu_s \tan\theta}\right)^{1/2} = 38.1 \text{ m s}^{-1}$$ ◀

5.11 SOLVING PROBLEMS IN MECHANICS

The three laws of motion that you have learnt in this chapter are the foundation of mechanics. You should now be able to handle a large variety of problems in mechanics. A typical problem in mechanics usually does not merely involve a single body under the action of given forces. More often, we will need to consider an assembly of different bodies exerting forces on each other. Besides, each body in the assembly experiences the force of gravity. When trying to solve a problem of this type, it is useful to remember the fact that we can choose any part of the assembly and apply the laws of motion to that part provided we include all forces on the chosen part due to the remaining parts of the assembly. We may call the chosen part of the assembly as the system and the remaining part of the assembly (plus any other agencies of forces) as the environment. We have followed the same method in solved examples. To handle a typical problem in mechanics systematically, one should use the following steps :

(i) Draw a diagram showing schematically the various parts of the assembly of bodies, the links, supports, etc.

(ii) Choose a convenient part of the assembly as one system.

(iii) Draw a separate diagram which shows this system and all the forces on the system by the remaining part of the assembly. Include also the forces on the system by other agencies. **Do not include the forces on the environment by the system**. A diagram of this type is known as 'a free-body diagram'. (Note this does not imply that the system under consideration is without a net force).

(iv) In a free-body diagram, include information about forces (their magnitudes and directions) that are either given or you are sure of (e.g., the direction of tension in a string along its length). The rest should be treated as unknowns to be determined using laws of motion.

(v) If necessary, follow the same procedure for another choice of the system. In doing so, employ Newton's third law. That is, if in the free-body diagram of *A*, the force on *A* due to *B* is shown as **F**, then in the free-body diagram of *B*, the force on *B* due to *A* should be shown as –**F**.

The following example illustrates the above procedure :

▶ ***Example 5.12*** See Fig. 5.15. A wooden block of mass 2 kg rests on a soft horizontal floor. When an iron cylinder of mass 25 kg is placed on top of the block, the floor yields steadily and the block and the cylinder together go down with an acceleration of 0.1 m s^{-2}. What is the action of the block on the floor (a) before and (b) after the floor yields ? Take g = 10 m s^{-2}. Identify the action-reaction pairs in the problem.

Answer

(a) The block is at rest on the floor. Its free-body diagram shows two forces on the block, the force of gravitational attraction by the earth equal to 2 × 10 = 20 N; and the normal force *R* of the floor on the block. By the First Law, the net force on the block must be zero i.e., *R* = 20 N. Using third law the action of the block (i.e. the force exerted on the floor by the block) is equal to 20 N and directed vertically downwards.

(b) The system (block + cylinder) accelerates downwards with 0.1 m s^{-2}. The free-body diagram of the system shows two forces on the system : the force of gravity due to the earth (270 N); and the normal force *R′* by the floor. Note, the free-body diagram of the system does not show the internal forces between the block and the cylinder. Applying the second law to the system,

$$270 - R' = 27 \times 0.1\text{N}$$

$$\text{ie. } R' = 267.3 \text{ N}$$

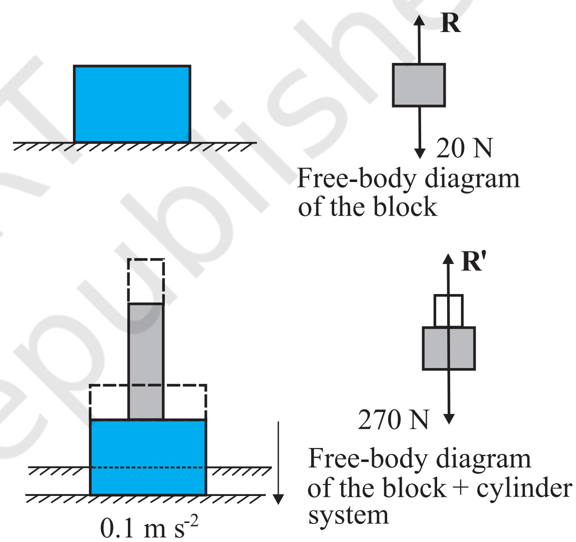

Fig. 5.15

By the third law, the action of the system on the floor is equal to 267.3 N vertically downward.

Action-reaction pairs

For (a): (i) the force of gravity (20 N) on the block by the earth (say, action); the force of gravity on the earth by the block (reaction) equal to 20 N directed upwards (not shown in the figure).
(ii) the force on the floor by the block (action); the force on the block by the floor (reaction).

For (b): (i) the force of gravity (270 N) on the system by the earth (say, action); the force of gravity on the earth by the system (reaction), equal to 270 N,

directed upwards (not shown in the figure).
(ii) the force on the floor by the system (action); the force on the system by the floor (reaction). In addition, for (b), the force on the block by the cylinder and the force on the cylinder by the block also constitute an action-reaction pair.

The important thing to remember is that an action-reaction pair consists of mutual forces which are always equal and opposite between two bodies. Two forces on the same body which happen to be equal and opposite can never constitute an action-reaction pair. The force of gravity on the mass in (a) or (b) and the normal force on the mass by the floor are not action-reaction pairs. These forces happen to be equal and opposite for (a) since the mass is at rest. They are not so for case (b), as seen already. The weight of the system is 270 N, while the normal force R' is 267.3 N. ◀

The practice of drawing free-body diagrams is of great help in solving problems in mechanics. It allows you to clearly define your system and consider all forces on the system due to objects that are not part of the system itself. A number of exercises in this and subsequent chapters will help you cultivate this practice.

SUMMARY

1. Aristotle's view that a force is necessary to keep a body in uniform motion is wrong. A force is necessary in practice to counter the opposing force of friction.
2. Galileo extrapolated simple observations on motion of bodies on inclined planes, and arrived at the law of inertia. Newton's first law of motion is the same law rephrased thus: *"Everybody continues to be in its state of rest or of uniform motion in a straight line, unless compelled by some external force to act otherwise"*. In simple terms, the First Law is **"If external force on a body is zero, its acceleration is zero"**.
3. Momentum (**p**) of a body is the product of its mass *(m)* and velocity (**v**) :
$$\mathbf{p} = m\mathbf{v}$$
4. Newton's second law of motion :
The rate of change of momentum of a body is proportional to the applied force and takes place in the direction in which the force acts. Thus
$$\mathbf{F} = k\frac{d\mathbf{p}}{dt} = k\,m\,\mathbf{a}$$
where **F** is the net external force on the body and **a** its acceleration. We set the constant of proportionality k = 1 in SI units. Then
$$\mathbf{F} = \frac{d\mathbf{p}}{dt} = m\mathbf{a}$$
The SI unit of force is newton : 1 N = 1 kg m s^{-2}.
(a) The second law is consistent with the First Law (**F** = 0 implies **a** = 0)
(b) It is a vector equation
(c) It is applicable to a particle, and also to a body or a system of particles, provided **F** is the total external force on the system and **a** is the acceleration of the system as a whole.
(d) **F** at a point at a certain instant determines **a** at the same point at that instant. That is the Second Law is a local law; **a** at an instant does not depend on the history of motion.
5. Impulse is the product of force and time which equals change in momentum.
The notion of impulse is useful when a large force acts for a short time to produce a measurable change in momentum. Since the time of action of the force is very short, one can assume that there is no appreciable change in the position of the body during the action of the impulsive force.
6. Newton's third law of motion:
To every action, there is always an equal and opposite reaction