* Concepts and formulas to Remember *

Concepts :- Quesion can be solve by both way direct formula and geometric approach which is venn diagram.

main formula :-

(PIE): Given X is finite set and $A_1, \dots A_n$ ben non-empty subsets of X then

$$|X - \bigcup_{i=1}^{n} A_i| = \sum_{I \subseteq [n]} (-1)^{|I|} |A_I|$$

where $A_I = \bigcap_{i \in I} A_i$

and $A_\phi \coloneqq X$.

where $[n] = \{1, 2, \dots n\}$.

above formula is used very rarly so only for $n=2$, and $n=3$ we have to know.

indirect formula can be say it is.

$n=2$

$$|X-(A \cup B)| = |X| - (|A| + |B| - |A \cap B|)$$

$$= |X| - |A| - |B| + |A \cap B|$$

$n=3$

$$|X - (A \cup B \cup C)| = |X| - (|A| + |B| + |C| - |B \cap A| - |B \cap C| - |C \cap A| + |C \cap A \cap B|)$$

$$= |X| - |A| - |B| - |C| + |A \cap B| + |B \cap C| + |C \cap A| - |A \cap B \cap C|$$

(PIE) principle of inclusion and Exclusion

direct formula:-

for $n=2$.

$$|A\cup B| = |A| + |B| - |A\cap B|$$

for $n=3$

$$|A\cup B\cup C| = |A| + |B| + |C| - |A\cap B| - |B\cap C| - |C\cap A| + |A\cap B\cap C|$$

Concepts and formula to remember.

formula and concept are same as given above.

but here we have some very good result's which we can derive from PIE.

1. no. of onto maps from set A to set B where $|A| = m$, $|B| = n$

$$= \sum_{i=0}^{n} (-1)^i \binom{n}{i} (n-i)^m$$

if $m = n$, we have $\underline{n!}$ ans.

2. total no. of dearrangment of n set.

is

$$d_n = n!\left(1 - 1 + \frac{1}{2} - \frac{1}{3!} + \frac{1}{4!} \cdots\cdots\right)$$

and d_n also follow reacuurance relation.

$$d_n = (n-1)(d_{n-1} + d_{n-2}).$$

3. $\phi(n)$ = total numbers of $x \le n$ such that $gcd(x,n) = 1$.

we have proved using PIE that

$$\phi(n) = \prod_{i=1}^{k} (1-P_k) ,$$

where $n = P_1 P_2 \cdots P_k$

and p_i are prime number for $i = 1$ to k.