

* Concepts and formulae to Remember *

Concepts:- Question can be solve by both way direct formula and geometric approach which is venn diagram.

main formula:-

(PIE): Given X is finite set and A_1, \dots, A_n be non-empty subsets of X then

$$\left| X - \bigcup_{i=1}^n A_i \right| = \sum_{I \subseteq [n]} (-1)^{|I|} |A_I|$$

where $A_I = \bigcap_{i \in I} A_i$

and $A_\emptyset = X$.

where $[n] = \{1, 2, \dots, n\}$.

above formula is used very rarely so only for $n=2$, and $n=3$ we have to know.

indirect formula can be say it is.

$$\begin{aligned} \underline{n=2} \quad |X - (A \cup B)| &= |X| - (|A| + |B| - |A \cap B|) \\ &= |X| - |A| - |B| + |A \cap B| \end{aligned}$$

$$\begin{aligned} \underline{n=3} \quad |X - (A \cup B \cup C)| &= |X| - (|A| + |B| + |C| - |B \cap A| - |B \cap C| \\ &\quad - |C \cap A| \\ &\quad + |C \cap A \cap B|) \\ &= |X| - |A| - |B| - |C| + |A \cap B| + |B \cap C| + |C \cap A| \\ &\quad - |A \cap B \cap C| \end{aligned}$$

(PIE) principle of inclusion and Exclusion
direct formula:-

for $n=2$.

$$|A \cup B| = |A| + |B| - |A \cap B|$$

for $n=3$

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |C \cap A| + |A \cap B \cap C|$$

Let X be a set of n elements. Let A_1, A_2, \dots, A_m be a collection of m subsets of X .

$$|X - \bigcap_{i=1}^m A_i| = \sum_{I \subseteq [m]} (-1)^{|I|} |A_I|$$

$$|A_I| = |A_1 \cap A_2 \cap \dots \cap A_m|$$

$$X = \phi A$$

$$|X - \bigcap_{i=1}^m A_i| = |X| - |A_1 \cap A_2 \cap \dots \cap A_m|$$

Let us find the number of elements in X which are not in any of the sets A_1, A_2, \dots, A_m .

Let A be a set of n elements.

$$|X - (A \cap B)| = |X| - (|A| + |B| - |A \cap B|) = |X| - |A| - |B| + |A \cap B|$$

$$|X - (A \cap B \cap C)| = |X| - (|A| + |B| + |C| - |A \cap B| - |B \cap C| - |C \cap A| + |A \cap B \cap C|)$$

$$= |X| - |A| - |B| - |C| + |A \cap B| + |B \cap C| + |C \cap A| - |A \cap B \cap C|$$