

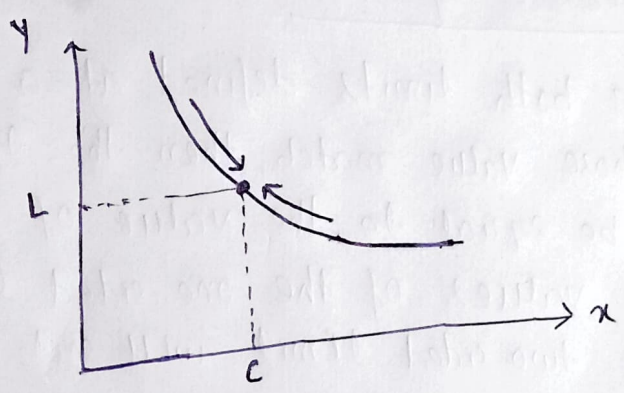
Notes - Limits

What is limit?

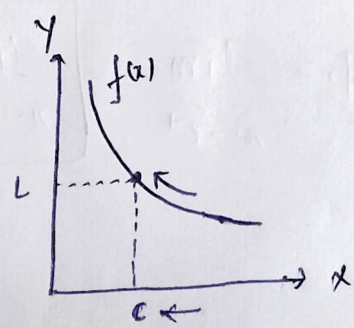
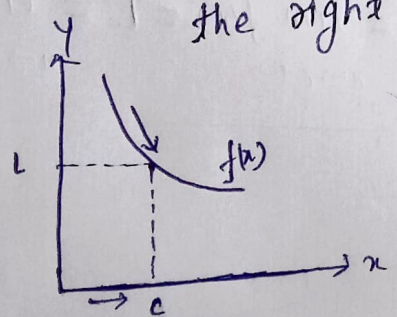
Limits Definition :- Let us consider a real-valued function f and the real number c , the limit is normally defined as

$$\lim_{x \rightarrow c} f(x) = L$$

- it is read as "the limit of f of x ($f(x)$), as x approaches c equals L ."
- The \lim shows limit, and fact that function $f(x)$ approaches c is described by the right arrow.

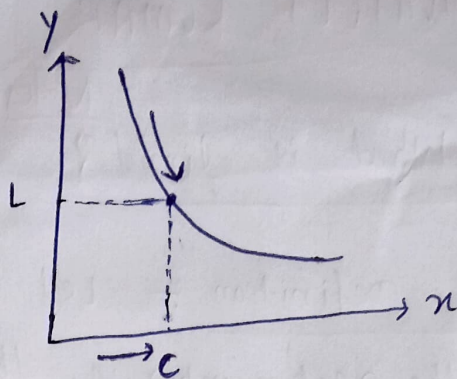


One Sided Limits - in calculus, a one-sided limit refers to either one of the two limits of a function $f(x)$ of a real variable x as x approaches a specified point either from the left or from the right.



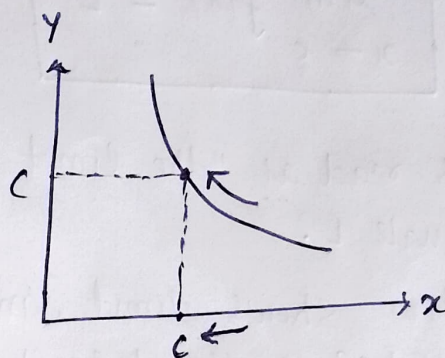
Left hand limit (LHL)

$$\lim_{x \rightarrow c^-} f(x) = L$$



Right hand limit (RHL)

$$\lim_{x \rightarrow c^+} f(x) = L$$



Condition for Existence of limit

→ if the function has both limits defined at a particular x value c and those values match, then the limit will exist and will be equal to the value of the one-sided limits. if the values of the one-sided limits do not match, then the two-sided limit will not exist.

→ For limit to exist at any value of x , say $x = c$ exists only when limit approaching from right of c and limit approaching from left of c are equal.

$$\lim_{x \rightarrow c^-} f(x) = \lim_{x \rightarrow c^+} f(x) = L = \lim_{x \rightarrow c} f(x)$$

Epsilon-delta definition of limit (ϵ - δ definition) ②

Let $f(x)$ be a function defined on an open interval around x_0 ($f(x_0)$ need not be defined). We say that the limit of $f(x)$ as x approaches x_0 is L , i.e.

$$\lim_{x \rightarrow x_0} f(x) = L$$

if for every $\epsilon > 0$ there exists $\delta > 0$ such that for all

$$x, \quad 0 < |x - x_0| < \delta \Rightarrow |f(x) - L| < \epsilon.$$

in other words, the definition states that we can make values returned by the function $f(x)$ as close as we would like to the value L by using only the points in a small enough interval around x_0 .

Properties of limits

The following is the list of properties of limits.

We assume that

$\lim_{x \rightarrow a} f(x)$ and $\lim_{x \rightarrow a} g(x)$ exist and c is constant.

$$\textcircled{1} \quad \lim_{x \rightarrow a} [c f(x)] = c \lim_{x \rightarrow a} f(x)$$

$$\textcircled{2} \quad \lim_{x \rightarrow a} [f(x) \pm g(x)] = \lim_{x \rightarrow a} f(x) \pm \lim_{x \rightarrow a} g(x)$$

$$\textcircled{3} \quad \lim_{x \rightarrow a} [f(x) \cdot g(x)] = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x)$$

$$\textcircled{4} \quad \lim_{x \rightarrow a} \left[\frac{f(x)}{g(x)} \right] = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}$$

$$\textcircled{5} \quad \lim_{x \rightarrow a} c = c$$

$$\textcircled{6} \quad \lim_{x \rightarrow a} x^n = a^n$$