

## Important formulas of Limits: -

$$\triangleright \lim_{x \rightarrow 0} (1+x)^{1/x} = e$$

$$\triangleright \lim_{x \rightarrow \infty} \left(1 + \frac{a}{x}\right)^x = e^a$$

$$\triangleright \lim_{x \rightarrow \infty} a^x = \begin{cases} \infty, & \text{if } a > 1 \\ 0, & \text{if } a < 1 \end{cases}$$

i.e.,  $a^\infty = \infty$ , if  $a > 1$  and  $a^\infty = 0$ , if  $a < 1$ .

$$\triangleright \lim_{x \rightarrow 0} \frac{(1+x)^n - 1}{x} = n$$

$$\triangleright \lim_{x \rightarrow 0} \frac{\sin^{-1} x}{x} = 1 = \lim_{x \rightarrow 0} \frac{\tan^{-1} x}{x}$$

$$\triangleright \lim_{x \rightarrow a} \sin^{-1} x = \sin^{-1} a, |a| \leq 1$$

$$\triangleright \lim_{x \rightarrow a} \cos^{-1} x = \cos^{-1} a, |a| \leq 1$$

$$\triangleright \lim_{x \rightarrow a} \tan^{-1} x = \tan^{-1} a, -\infty < a < \infty$$

$$\triangleright \lim_{x \rightarrow e} \log_e x = 1$$

$$\triangleright \text{If } \lim_{x \rightarrow a} f(x) = +\infty, \text{ then } \lim_{x \rightarrow a} \frac{1}{f(x)} = 0$$

Let  $\lim_{x \rightarrow a} f(x) = l$  and  $\lim_{x \rightarrow a} g(x) = m$ , then  $\lim_{x \rightarrow a} (f(x))^{g(x)} = l^m$

$\triangleright$  If  $f(x) \leq g(x)$  for every  $x$  in the deleted neighbourhood (nbd) of  $a$ , then  $\lim_{x \rightarrow a} f(x) \leq \lim_{x \rightarrow a} g(x)$ .

$\triangleright$  If  $f(x) \leq g(x) \leq h(x)$  for every  $x$  in the deleted nbd of  $a$  and  $\lim_{x \rightarrow a} f(x) = l = \lim_{x \rightarrow a} h(x)$ , then  $\lim_{x \rightarrow a} g(x) = l$ .

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**Theorems on Limits:** If  $f(x)$  and  $g(x)$  are two functions, then

$$(i) \lim_{x \rightarrow a} [f(x) \pm g(x)] = \lim_{x \rightarrow a} f(x) \pm \lim_{x \rightarrow a} g(x)$$

$$(ii) \lim_{x \rightarrow a} [f(x) \cdot g(x)] = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x)$$

$$(iii) \lim_{x \rightarrow a} \left[ \frac{f(x)}{g(x)} \right] = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)} \text{ if } \lim_{x \rightarrow a} g(x) \neq 0.$$

$$(iv) \lim_{x \rightarrow a} [k f(x)] = \lim_{x \rightarrow a} f(x), \text{ where } k \text{ is constant.}$$

$$(v) \lim_{x \rightarrow a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \rightarrow a} f(x)}$$

$$(vi) \lim_{x \rightarrow a} (f(x))^{p/q} = \left( \lim_{x \rightarrow a} f(x) \right)^{p/q} \text{ where } p \text{ and } q \text{ are integers.}$$

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