Question: -

Let $f(x) = \dfrac{1 - x\,(1 + |1 - x|)}{|1 - x|} \cos\left(\dfrac{1}{1 - x}\right)$

for $x \neq 1$. Then

(a) $\lim_{x \to 1^+} f(x) = 0$

(b) $\lim_{x \to 1^-} f(x)$ does not exist

(c) $\lim_{x \to 1^-} f(x) = 0$

(d) $\lim_{x \to 1^+} f(x)$ does not exist

Solution: -

$$f(x) = \frac{1 - x(1 + |1 - x|)}{|1 - x|} \cos\left(\frac{1}{1 - x}\right)$$

Now, $\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^-} \dfrac{1 - x(1 + 1 - x)}{1 - x} \cos\left(\dfrac{1}{1 - x}\right)$

$$= \lim_{x \to 1^-} (1 - x) \cos\left(\frac{1}{1 - x}\right) = 0$$

and $\lim\limits_{x \to 1^+} f(x) = \lim\limits_{x \to 1^+} \dfrac{1 - x(1 - 1 + x)}{x - 1} \cos\left(\dfrac{1}{1 - x}\right)$

$= \lim\limits_{x \to 1^+} -(x + 1) \cdot \cos\left(\dfrac{1}{x + 1}\right)$, which does not exist.