Question: -

$$\lim_{x \to 0} \frac{x \tan 2x - 2x \tan x}{(1 - \cos 2x)^2} \text{ is}$$

(1999, 2M)

(a) 2

(b) –2

(c) $\dfrac{1}{2}$

(d) $-\dfrac{1}{2}$

Solution: -

$$\lim_{x \to 0} \frac{x \tan 2x - 2x \tan x}{(1 - \cos 2x)^2}$$

NOTE In trigonometry try to make all trigonometric functions in same angle. It is called 3rd Golden rule of trigonometry.

$$= \lim_{x \to 0} \frac{x \dfrac{2 \tan x}{1 - \tan^2 x} - 2x \tan x}{(2 \sin^2 x)^2}$$

$$= \lim_{x \to 0} \frac{2x \tan x \left[\dfrac{1}{1 - \tan^2 x} - 1 \right]}{4 \sin^4 x}$$

$$= \lim_{x \to 0} \frac{2x \tan x \left[\dfrac{1 - 1 + \tan^2 x}{1 - \tan^2 x} \right]}{4 \sin^4 x}$$

$$= \lim_{x \to 0} \frac{2x \tan^3 x}{2 \sin^4 x (1 - \tan^2 x)} = \lim_{x \to 0} \frac{1}{2} \frac{x \left(\dfrac{\tan x}{x} \right)^3 \cdot x^3}{\sin^4 x (1 - \tan^2 x)}$$

$$= \frac{1}{2} \lim_{x \to 0} \frac{\left(\dfrac{\tan x}{x} \right)^3}{\left(\dfrac{\sin x}{x} \right)^4 (1 - \tan^2 x)} = \frac{1 \cdot (1)^3}{2(1)^4 (1 - 0)} = \frac{1}{2}$$