Question: -

$$\lim_{x \to 0} \frac{x + 2\sin x}{\sqrt{x^2 + 2\sin x + 1} - \sqrt{\sin^2 x - x + 1}} \text{ is}$$

(2019 Main, 12 April II)

(a) 6 (b) 2 (c) 3 (d) 1

Solution: -

Let

$$P = \lim_{x \to 0} \frac{x + 2\sin x}{\sqrt{x^2 + 2\sin x + 1} - \sqrt{\sin^2 x - x + 1}} \qquad \left[\frac{0}{0} \text{ form}\right]$$

On rationalization, we get

$$P = \lim_{x \to 0} \frac{(x + 2\sin x)}{x^2 + 2\sin x + 1 - \sin^2 x + x - 1} \times \left(\sqrt{x^2 + 2\sin x + 1} + \sqrt{\sin^2 x - x + 1}\right)$$

$$= \lim_{x \to 0} \left(\sqrt{x^2 + 2\sin x + 1} + \sqrt{\sin^2 x - x + 1}\right) \times \lim_{x \to 0} \frac{x + 2\sin x}{x^2 - \sin^2 x + 2\sin x + x}$$

$$= 2 \times \lim_{x \to 0} \frac{x + 2\sin x}{x^2 - \sin^2 x + 2\sin x + x} \qquad \left[\frac{0}{0} \text{ form}\right]$$

Now applying the L′ Hopital's rule, we get

$$P = 2 \times \lim_{x \to 0} \frac{1 + 2\cos x}{2x - \sin 2x + 2\cos x + 1}$$

$$= 2\,\frac{(1 + 2)}{0 - 0 + 2 + 1} \qquad \text{[on applying limit]}$$

$$= 2 \times \frac{3}{3} = 2$$

$$\Rightarrow \lim_{x \to 0} \frac{x + 2\sin x}{\sqrt{x^2 + 2\sin x + 1} - \sqrt{\sin^2 x - x + 1}} = 2$$