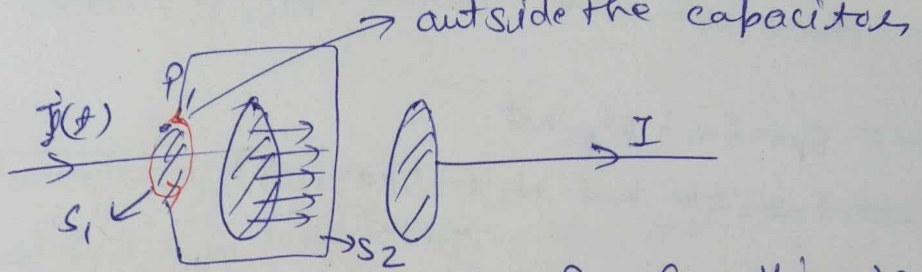
Displacement current :-

A changing electric field gives rise to a magnetic field,
Let us consider the process of charging of a capacitor
& apply Ampere's circuital law given by

$$\boxed{\oint \vec{B}\cdot d\vec{l} = \mu_0 I(t)}$$



what is magnetic field at point P_1. For this we
draw a circuital loop at P_1

→ We consider a plane circular loop of radius
or whose plane is per. to the direction of current
carrying wire.

For surface S_1, $I_{enclosed} = I$

For surface S_2, $I_{enc} = 0$

→ Now we calculate electric flux through S_2

$$\Rightarrow \quad \phi_E = EA = \frac{\sigma}{\varepsilon_0} A = \frac{Q}{\varepsilon_0}$$

A → area of plate
Q → total charge on plate

$\therefore$ current $I = \frac{dQ}{dt}$

$= \varepsilon_0 \frac{d\phi_E}{dt}$

So if we write:

Conduction current

$$\boxed{\oint \vec{B}\cdot d\vec{l} = \mu_0 I_c + \mu_0 \varepsilon_0 \frac{d\phi_E}{dt}}$$

add to modified ampere law

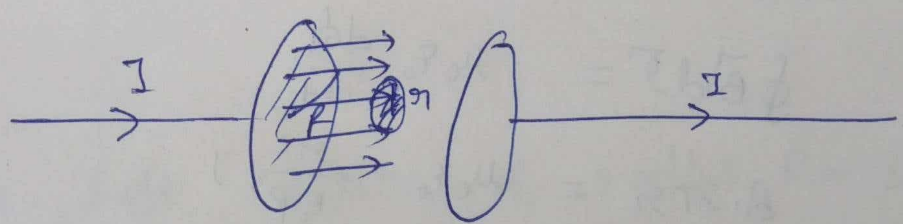
Displacement current

$$\therefore \boxed{I_D = \varepsilon_0 \frac{d\phi_E}{dt}}$$

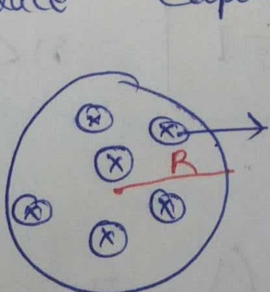
$$\Rightarrow \boxed{\oint \vec{B}\cdot d\vec{l} = \mu_0 (I_c + I_D)}$$

Displacement current density $\boxed{J_D = \varepsilon_0 \frac{dE}{dt}}$

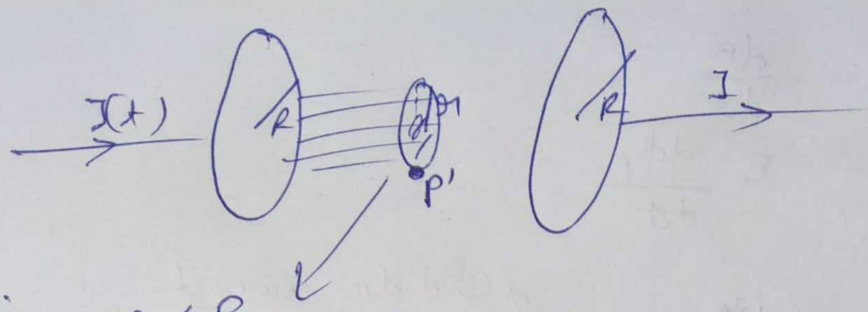
e.g.



parallel place capacitor with circular plate



I want to calculate electric field at some point of plate.



(i) $r<R$

$\oint \vec{B}\cdot d\vec{l} = B\cdot 2\pi r$

$\phi_E = \pi r^2 E = \pi r^2 \dfrac{\sigma}{\varepsilon_0}$ → surface charge density

$= \pi r^2 \cdot \dfrac{1}{\pi R^2} Q$ $\boxed{\sigma = \dfrac{Q}{\pi R^2}}$ for plate

$= \dfrac{Q r^2}{\varepsilon_0 R^2}$

$\therefore \dfrac{d\phi_E}{dt} = \dfrac{r^2}{\varepsilon_0 R^2}\cdot\dfrac{dQ}{dt}$

$$\boxed{\frac{d\phi_E}{dt} = \frac{r^2}{\varepsilon_0 R^2} I}$$

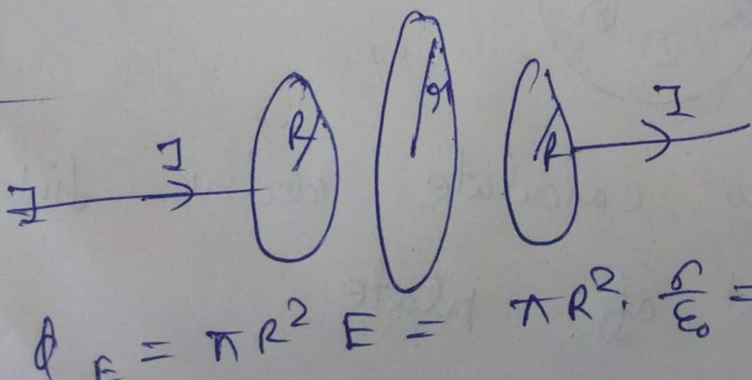
For surface P' :- $I_C = 0$

$\therefore \oint \vec{B}\cdot d\vec{l} = \mu_0\varepsilon_0 \dfrac{d\phi_E}{dt}$

$B\cdot 2\pi r = \mu_0\varepsilon_0 \cdot \dfrac{r^2}{\varepsilon_0 R^2} I$

$$\boxed{B = \frac{\mu_0 r}{2\pi R^2} I} \quad r<R$$

(ii) $r>R$



Now $\phi_E = \pi R^2 E = \pi R^2 \dfrac{\sigma}{\varepsilon_0} = \pi R^2 \dfrac{Q}{\pi R^2 \varepsilon_0}$

$$\frac{d\phi_E}{dt} = \frac{1}{\varepsilon_0}\frac{dq}{dt} = \frac{1}{\varepsilon_0} I$$

from Modifi. Ampere's law,

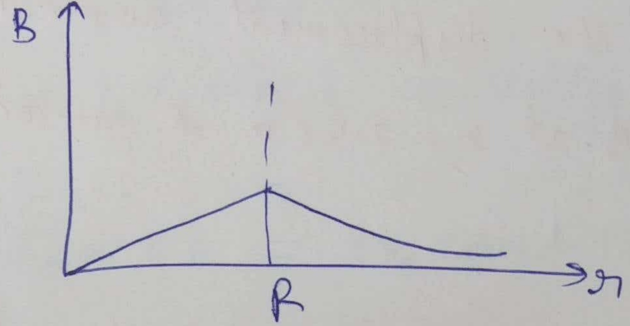
$$\therefore \oint \vec{B}\cdot d\vec{l} = \mu_0 \underset{0}{I_c} + \mu_0\varepsilon_0 \frac{d\phi_E}{dt}$$

$$= \mu_0 \cancel{\varepsilon_0} \frac{I}{\cancel{\varepsilon_0}}$$

$$B \cdot 2\pi r = \mu_0 I$$

$$\boxed{B = \frac{\mu_0 I}{2\pi r}} \qquad r > R$$

$\Rightarrow$



Magnetic field is continuous at $r = R$

$$\boxed{B(r=R) = \frac{\mu_0 I}{2\pi R}}$$

e.g. We have capacitor of radius $R = 1\,cm$

$I = 1\,A$

then for $r < R$ Let $r = 0.5\,cm$

$$B = \frac{\mu_0 r}{2\pi R^2} I$$

$$= \frac{4\pi \times 10^{-7} \times 0.5 \times 10^{-2} \times 1}{2\pi \times 10^{-4}}$$

$$\boxed{B = 10\,\mu T}$$

for $r > R$ $\quad r = 5\,cm$

$$B = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} \times 1}{2\pi \times 5 \times 10^{-2}}$$

$$= 4\,\mu T$$

problem: A parallel plate capacitor (air filled) is getting charged and current at a specific time is 0.45 A.

If radius of the plate $R = 5\,cm$.

a) Calculate the total displacement current between the capacitor plate

b) Calculate the displacement current density

c) Calculate B at $r = 2.5\,cm$ & $r = 10\,cm$.

Solve this:

Faraday's law:-

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\phi_B}{dt} = -\frac{d}{dt}\int \vec{B} \cdot d\vec{A} \quad \text{—(i)}$$

Modified Ampere law (with $I_c = 0$)

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \epsilon_0 \frac{d\phi_E}{dt} = \mu_0 \epsilon_0 \frac{d}{dt}\int \vec{E} \cdot d\vec{l} \quad \text{—(ii)}$$

(i) ⇒ changing magnetic field produces electric field.

(ii) ⇒ time varying electric field produces magnetic field.

→ Conduction current density $\boxed{J_c = \underbrace{\sigma}_{\text{conductivity of medium}} E}$

→ Displacement current density $\boxed{J_D = \epsilon_0 \frac{dE}{dt}}$ (freespace)

$\boxed{J_D = \epsilon \frac{dE}{dt}}$ (for medium)

$\boxed{\epsilon = \epsilon_0 \underbrace{K}_{\text{dielectric constant}}}$

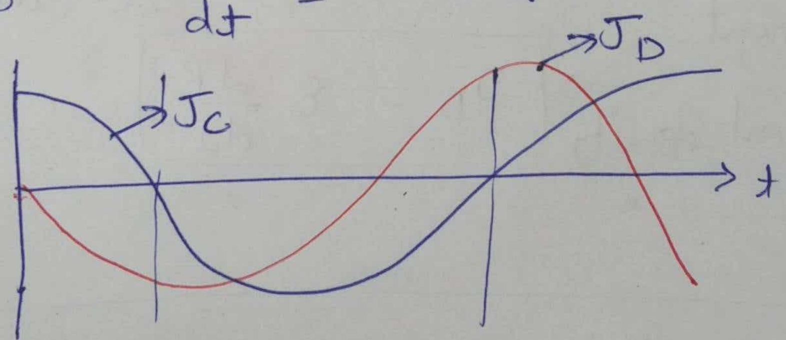
Good conductor,

~~$\sigma \approx 10^7$~~

Let $E = E_0 \cos\omega t$ (oscillating electric field)

then $J_c = \sigma E = \sigma E_0 \cos\omega t$

& $J_D = \epsilon \frac{dE}{dt} = -\epsilon \omega E_0 \sin\omega t$



there is phase difference b/w J_c & J_D.

Now $J_c(max) = \sigma E_0$

$J_D(max) = +\epsilon \omega E_0$

$$\boxed{\left.\frac{J_D}{J_c}\right|_{max} = \frac{\epsilon \omega E_0}{\sigma E_0} = \frac{\epsilon \omega}{\sigma} = \frac{2\pi \nu \epsilon}{\sigma}}$$