Chapter Eight

ELECTROMAGNETIC WAVES

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8.1 Introduction

In Chapter 4, we learnt that an electric current produces magnetic field and that two current-carrying wires exert a magnetic force on each other. Further, in Chapter 6, we have seen that a magnetic field changing with time gives rise to an electric field. Is the converse also true? Does an electric field changing with time give rise to a magnetic field? James Clerk Maxwell (1831-1879), argued that this was indeed the case – not only an electric current but also a time-varying electric field generates magnetic field. While applying the Ampere's circuital law to find magnetic field at a point outside a capacitor connected to a time-varying current, Maxwell noticed an inconsistency in the Ampere's circuital law. He suggested the existence of an additional current, called by him, the displacement current to remove this inconsistency.

Maxwell formulated a set of equations involving electric and magnetic fields, and their sources, the charge and current densities. These equations are known as Maxwell's equations. Together with the Lorentz force formula (Chapter 4), they mathematically express all the basic laws of electromagnetism.

The most important prediction to emerge from Maxwell's equations is the existence of electromagnetic waves, which are (coupled) time-varying electric and magnetic fields that propagate in space. The speed of the waves, according to these equations, turned out to be very close to

JAMES CLERK MAXWELL (1831–1879)

James Clerk Maxwell (1831 – 1879) Born in Edinburgh, Scotland, was among the greatest physicists of the nineteenth century. He derived the thermal velocity distribution of molecules in a gas and was among the first to obtain reliable estimates of molecular parameters from measurable quantities like viscosity, etc. Maxwell's greatest acheivement was the unification of the laws of electricity and magnetism (discovered by Coulomb, Oersted, Ampere and Faraday) into a consistent set of equations now called Maxwell's equations. From these he arrived at the most important conclusion that light is an electromagnetic wave. Interestingly, Maxwell did not agree with the idea (strongly suggested by the Faraday's laws of electrolysis) that electricity was particulate in nature.

the speed of light(3×10^8 m/s), obtained from optical measurements. This led to the remarkable conclusion that light is an electromagnetic wave. Maxwell's work thus unified the domain of electricity, magnetism and light. Hertz, in 1885, experimentally demonstrated the existence of electromagnetic waves. Its technological use by Marconi and others led in due course to the revolution in communication that we are witnessing today.

In this chapter, we first discuss the need for displacement current and its consequences. Then we present a descriptive account of electromagnetic waves. The broad spectrum of electromagnetic waves, stretching from γ rays (wavelength ~10^{-12} m) to long radio waves (wavelength ~10^6 m) is described. How the electromagnetic waves are sent and received for communication is discussed in Chapter 15.

8.2 DISPLACEMENT CURRENT

We have seen in Chapter 4 that an electrical current produces a magnetic field around it. Maxwell showed that for logical consistency, a changing electric field *must also* produce a magnetic field. This effect is of great importance because it explains the existence of radio waves, gamma rays and visible light, as well as all other forms of electromagnetic waves.

To see how a changing electric field gives rise to a magnetic field, let us consider the process of charging of a capacitor and apply Ampere's circuital law given by (Chapter 4)

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \, i(t) \qquad (8.1)$$

to find magnetic field at a point outside the capacitor. Figure 8.1(a) shows a parallel plate capacitor C which is a part of circuit through which a time-dependent current $i(t)$ flows . Let us find the magnetic field at a point such as P, in a region outside the parallel plate capacitor. For this, we consider a plane circular loop of radius r whose plane is perpendicular to the direction of the current-carrying wire, and which is centred symmetrically with respect to the wire [Fig. 8.1(a)]. From symmetry, the magnetic field is directed along the circumference of the circular loop and is the same in magnitude at all points on the loop so that if B is the magnitude of the field, the left side of Eq. (8.1) is $B(2\pi r)$. So we have

$$B(2\pi r) = \mu_0 i(t) \qquad (8.2)$$

Now, consider a different surface, which has the same boundary. This is a pot like surface [Fig. 8.1(b)] which nowhere touches the current, but has its bottom between the capacitor plates; its mouth is the circular loop mentioned above. Another such surface is shaped like a tiffin box (without the lid) [Fig. 8.1(c)]. On applying Ampere's circuital law to such surfaces with the *same* perimeter, we find that the left hand side of Eq. (8.1) has not changed but the right hand side is *zero* and *not* $\mu_0 i$, since *no* current passes through the surface of Fig. 8.1(b) and (c). So we have a *contradiction*; calculated one way, there is a magnetic field at a point P; calculated another way, the magnetic field at P is zero.
Since the contradiction arises from our use of Ampere's circuital law, this law must be missing something. The missing term must be such that one gets the same magnetic field at point P, no matter what surface is used.

We can actually guess the missing term by looking carefully at Fig. 8.1(c). Is there anything passing through the surface S *between* the plates of the capacitor? Yes, of course, the electric field! If the plates of the capacitor have an area *A*, and a total charge *Q*, the magnitude of the electric field **E** between the plates is $(Q/A)/\varepsilon_0$ (see Eq. 2.41). The field is perpendicular to the surface *S* of Fig. 8.1(c). It has the same magnitude over the area *A* of the capacitor plates, and vanishes outside it. So what is the *electric* flux Φ_E through the surface *S* ? Using Gauss's law, it is

$$\Phi_E = |\mathbf{E}|A = \frac{1}{\varepsilon_0}\frac{Q}{A}A = \frac{Q}{\varepsilon_0} \tag{8.3}$$

Now if the charge *Q* on the capacitor plates changes with time, there is a current $i = (dQ/dt)$, so that using Eq. (8.3), we have

$$\frac{d\Phi_E}{dt} = \frac{d}{dt}\left(\frac{Q}{\varepsilon_0}\right) = \frac{1}{\varepsilon_0}\frac{dQ}{dt}$$

This implies that for consistency,

$$\varepsilon_0\left(\frac{d\Phi_E}{dt}\right) = i \tag{8.4}$$

This is the missing term in Ampere's circuital law. If we generalise this law by adding to the total current carried by conductors through the surface, another term which is ε_0 times the rate of change of electric flux through the same surface, the *total* has the same value of current *i* for all surfaces. If this is done, there is no contradiction in the value of *B* obtained anywhere using the generalised Ampere's law. *B* at the point P is non-zero no matter which surface is used for calculating it. *B* at a point P outside the plates [Fig. 8.1(a)] is the same as at a point M just inside, as it should be. The current carried by conductors due to flow of charges is called *conduction current*. The current, given by Eq. (8.4), is a new term, and is due to changing electric field (or electric *displacement*, an old term still used sometimes). It is, therefore, called *displacement current* or Maxwell's displacement current. Figure 8.2 shows the electric and magnetic fields inside the parallel plate capacitor discussed above.

The generalisation made by Maxwell then is the following. The source of a magnetic field is not *just* the conduction electric current due to flowing

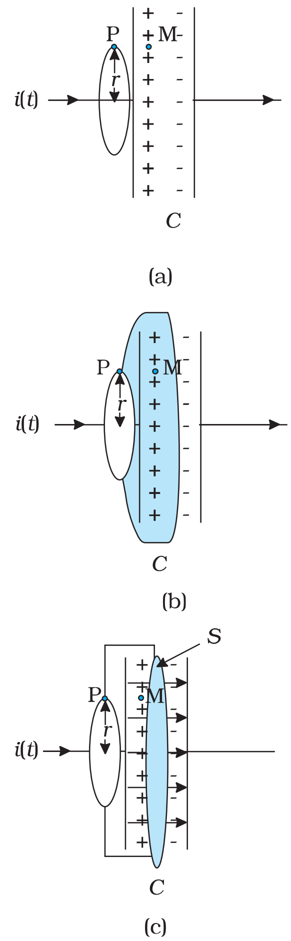

FIGURE 8.1 A parallel plate capacitor *C*, as part of a circuit through which a time dependent current *i* (*t*) flows, (a) a loop of radius *r*, to determine magnetic field at a point P on the loop; (b) a pot-shaped surface passing through the interior between the capacitor plates with the loop shown in (a) as its rim; (c) a tiffin-shaped surface with the circular loop as its rim and a flat circular bottom *S* between the capacitor plates. The arrows show uniform electric field between the capacitor plates.

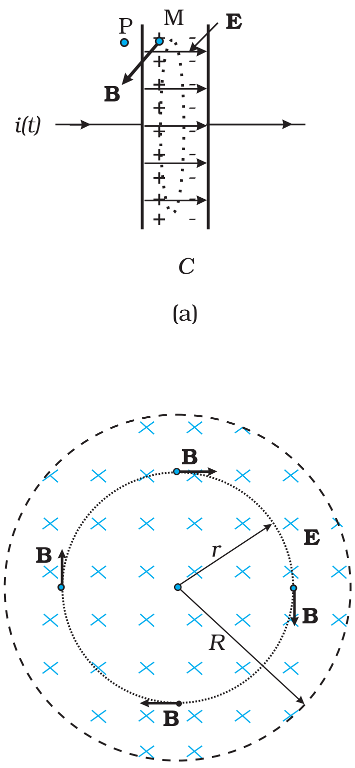

FIGURE 8.2 (a) The electric and magnetic fields **E** and **B** between the capacitor plates, at the point M. (b) A cross sectional view of Fig. (a).

charges, but also the time rate of change of electric field. More precisely, the total current i is the sum of the conduction current denoted by i_c, and the displacement current denoted by i_d $(= \varepsilon_0\, (d\Phi_E/dt))$. So we have

$$i = i_c + i_d = i_c + \varepsilon_0 \frac{d\Phi_E}{dt} \tag{8.5}$$

In explicit terms, this means that outside the capacitor plates, we have only conduction current $i_c = i$, and no displacement current, i.e., $i_d = 0$. On the other hand, inside the capacitor, there is no conduction current, i.e., $i_c = 0$, and there is only displacement current, so that $i_d = i$.

The generalised (and correct) Ampere's circuital law has the same form as Eq. (8.1), with one difference: "the *total current* passing through any surface of which the closed loop is the perimeter" is the sum of the conduction current and the displacement current. The generalised law is

$$\oint \mathbf{B}\cdot d\mathbf{l} = \mu_0\, i_c + \mu_0\, \varepsilon_0 \frac{d\Phi_E}{dt} \tag{8.6}$$

and is known as Ampere-Maxwell law.

In all respects, the displacement current has the same physical effects as the conduction current. In some cases, for example, steady electric fields in a conducting wire, the displacement current may be zero since the electric field **E** does not change with time. In other cases, for example, the charging capacitor above, both conduction and displacement currents may be present in different regions of space. In most of the cases, they both may be present in the same region of space, as there exist no perfectly conducting or perfectly insulating medium. Most interestingly, there may be large regions of space where there is *no* conduction current, but there is only a displacement current due to time-varying electric fields. In such a region, we expect a magnetic field, though there is no (conduction) current source nearby! The prediction of such a displacement current can be verified experimentally. For example, a *magnetic* field (say at point M) between the plates of the capacitor in Fig. 8.2(a) can be measured and is seen to be the same as that just outside (at P).

The displacement current has (literally) far reaching consequences. One thing we immediately notice is that the laws of electricity and magnetism are now more symmetrical*. Faraday's law of induction states that there is an induced emf *equal to the rate of* change of magnetic flux. Now, since the emf between two points 1 and 2 is the work done per unit charge in taking it from 1 to 2, the existence of an emf implies the existence of an electric field. So, we can rephrase Faraday's law of electromagnetic induction by saying that a *magnetic field*, changing with time, gives rise to an *electric field*. Then, the fact that an *electric field* changing with time gives rise to a *magnetic field*, is the symmetrical counterpart, and is

* They are still not perfectly symmetrical; there are no known sources of magnetic field (magnetic monopoles) analogous to electric charges which are sources of electric field.

a consequence of the displacement current being a source of a magnetic field. Thus, time- dependent electric and magnetic fields give rise to each other! Faraday's law of electromagnetic induction and Ampere-Maxwell law give a quantitative expression of this statement, with the current being the total current, as in Eq. (8.5). One very important consequence of this symmetry is the existence of electromagnetic waves, which we discuss qualitatively in the next section.

MAXWELL'S EQUATIONS IN VACUUM

1. $\oint \mathbf{E} \cdot d\mathbf{A} = Q/\varepsilon_0$ (Gauss's Law for electricity)

2. $\oint \mathbf{B} \cdot d\mathbf{A} = 0$ (Gauss's Law for magnetism)

3. $\oint \mathbf{E} \cdot d\mathbf{l} = \dfrac{-d\Phi_B}{dt}$ (Faraday's Law)

4. $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 i_c + \mu_0 \varepsilon_0 \dfrac{d\Phi_E}{dt}$ (Ampere – Maxwell Law)

Example 8.1 A parallel plate capacitor with circular plates of radius 1 m has a capacitance of 1 nF. At $t = 0$, it is connected for charging in series with a resistor $R = 1\ \text{M}\Omega$ across a 2V battery (Fig. 8.3). Calculate the magnetic field at a point P, halfway between the centre and the periphery of the plates, after $t = 10^{-3}$ s. (The charge on the capacitor at time t is $q(t) = CV[1 - \exp(-t/\tau)]$, where the time constant τ is equal to CR.)

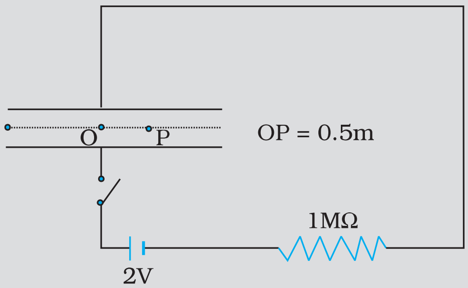

FIGURE 8.3

Solution The time constant of the CR circuit is $\tau = CR = 10^{-3}$ s. Then, we have

$q(t) = CV[1 - \exp(-t/\tau)]$

$= 2 \times 10^{-9}\ [1 - \exp(-t/10^{-3})]$

The electric field in between the plates at time t is

$E = \dfrac{q(t)}{\varepsilon_0 A} = \dfrac{q}{\pi\varepsilon_0}$; $A = \pi(1)^2\ \text{m}^2$ = area of the plates.

Consider now a circular loop of radius (1/2) m parallel to the plates passing through P. The magnetic field **B** at all points on the loop is

EXAMPLE 8.1

along the loop and of the same value.
The flux Φ_E through this loop is
Φ_E = $E \times$ area of the loop

$$= E \times \pi \times \left(\frac{1}{2}\right)^2 = \frac{\pi E}{4} = \frac{q}{4\varepsilon_0}$$

The displacement current

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \frac{1}{4}\frac{dq}{dt} = 0.5 \times 10^{-6} \exp(-1)$$

at $t = 10^{-3}$s. Now, applying Ampere-Maxwell law to the loop, we get

$$B \times 2\pi \times \left(\frac{1}{2}\right) = \mu_0 (i_c + i_d) = \mu_0 (0 + i_d) = 0.5\times10^{-6}\ \mu_0 \exp(-1)$$

or, $B = 0.74 \times 10^{-13}$ T

EXAMPLE 8.1

8.3 ELECTROMAGNETIC WAVES

8.3.1 Sources of electromagnetic waves

How are electromagnetic waves produced? Neither stationary charges nor charges in uniform motion (steady currents) can be sources of electromagnetic waves. The former produces only electrostatic fields, while the latter produces magnetic fields that, however, do not vary with time. It is an important result of Maxwell's theory that accelerated charges radiate electromagnetic waves. The proof of this basic result is beyond the scope of this book, but we can accept it on the basis of rough, qualitative reasoning. Consider a charge oscillating with some frequency. (An oscillating charge is an example of accelerating charge.) This produces an oscillating electric field in space, which produces an oscillating magnetic field, which in turn, is a source of oscillating electric field, and so on. The oscillating electric and magnetic fields thus regenerate each other, so to speak, as the wave propagates through the space. The frequency of the electromagnetic wave naturally equals the frequency of oscillation of the charge. The energy associated with the propagating wave comes at the expense of the energy of the source – the accelerated charge.

From the preceding discussion, it might appear easy to test the prediction that light is an electromagnetic wave. We might think that all we needed to do was to set up an ac circuit in which the current oscillate at the frequency of visible light, say, yellow light. But, alas, that is not possible. The frequency of yellow light is about 6×10^{14} Hz, while the frequency that we get even with modern electronic circuits is hardly about 10^{11} Hz. This is why the experimental demonstration of electromagnetic wave had to come in the low frequency region (the radio wave region), as in the Hertz's experiment (1887).

Hertz's successful experimental test of Maxwell's theory created a sensation and sparked off other important works in this field. Two important achievements in this connection deserve mention. Seven years after Hertz, Jagdish Chandra Bose, working at Calcutta (now Kolkata),