Question: -

A straight line through the vertex P of a ΔPQR intersects the side QR at the point S and the circumcircle of the ΔPQR at the point T. If S is not the centre of the circumcircle, then **(2008, 4M)**

(a) $\frac{1}{PS}+\frac{1}{ST}<\frac{2}{\sqrt{QS\times SR}}$ (b) $\frac{1}{PS}+\frac{1}{ST}>\frac{2}{\sqrt{QS\times SR}}$

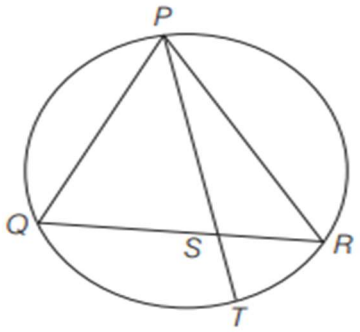
(c) $\frac{1}{PS}+\frac{1}{ST}<\frac{4}{QR}$ (d) $\frac{1}{PS}+\frac{1}{ST}>\frac{4}{QR}$

Solution: -

Let a straight line through the vertex P of a given ΔPQR intersects the side QR at the point S and the circumcircle of ΔPQR at the point T.

Points P, Q, R, T are concyclic, then $PS\cdot ST = QS\cdot SR$

Now, $\frac{PS+ST}{2}>\sqrt{PS\cdot ST}$ [$\because$ AM > GM]



and $\frac{1}{PS}+\frac{1}{ST}>\frac{2}{\sqrt{PS\cdot ST}}=\frac{2}{\sqrt{QS\cdot SR}}$

Also, $\frac{SQ+QR}{2}>\sqrt{SQ\cdot SR}$

$\Rightarrow \frac{QR}{2}>\sqrt{SQ\cdot SR}$

$\Rightarrow \frac{1}{\sqrt{SQ\cdot SR}}>\frac{2}{QR}$

$\Rightarrow \frac{2}{\sqrt{SQ\cdot SR}}>\frac{4}{QR}$

$\therefore \frac{1}{PS}+\frac{1}{ST}>\frac{2}{\sqrt{QS\cdot SR}}>\frac{4}{QR}$