**Question: -**

In a $\Delta PQR$, let $\angle PQR = 30°$ and the sides $PQ$ and $QR$ have lengths $10\sqrt{3}$ and 10, respectively. Then, which of the following statement(s) is (are) TRUE? **(2018 Adv)**

(a) $\angle QPR = 45°$

(b) The area of the $\Delta PQR$ is $25\sqrt{3}$ and $\angle QRP = 120°$

(c) The radius of the incircle of the $\Delta PQR$ is $10\sqrt{3} - 15$

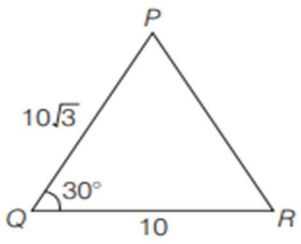
(d) The area of the circumcircle of the $\Delta PQR$ is $100\ \pi$

**Solution**: -

We have,

In $\Delta PQR$

$$\angle PQR = 30°$$
$$PQ = 10\sqrt{3}$$
$$QR = 10$$




By cosine rule

$$\cos 30° = \frac{PQ^2 + QR^2 - PR^2}{2PQ \cdot QR}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{300 + 100 - PR^2}{200\sqrt{3}}$$

$$\Rightarrow \quad 300 = 300 + 100 - PR^2$$

$$\Rightarrow \quad PR = 10$$

Since, $PR = QR = 10$

$$\therefore \quad \angle QPR = 30° \text{ and } \angle QRP = 120°$$

$$\text{Area of } \Delta PQR = \frac{1}{2} PQ \cdot QR \cdot \sin 30°$$

$$= \frac{1}{2} \times 10\sqrt{3} \times 10 \times \frac{1}{2} = 25\sqrt{3}$$

Radius of incircle of

$$\Delta PQR = \frac{\text{Area of } \Delta PQR}{\text{Semi - perimetre of } \Delta PQR}$$

i.e. $$r = \frac{\Delta}{s} = \frac{25\sqrt{3}}{\frac{10\sqrt{3} + 10 + 10}{2}} = \frac{25\sqrt{3}}{5(\sqrt{3} + 2)}$$

$$\Rightarrow \quad r = 5\sqrt{3}(2 - \sqrt{3})$$

$$= 10\sqrt{3} - 15$$

and radius of circumcircle (R) $= \frac{abc}{4\Delta} = \frac{10\sqrt{3} \times 10 \times 10}{4 \times 25\sqrt{3}} = 10$

$\therefore$ Area of circumcircle of

$$\Delta PQR = \pi R^2 = 100\ \pi$$

Hence, option (b), (c) and (d) are correct answer.