Question: -

Internal bisector of $\angle A$ of ΔABC meets side BC at D. A line drawn through D perpendicular to AD intersects the side AC at E and side AB at F. If a, b, c represent sides of ΔABC, then **(2006, 5M)**

(a) AE is HM of b and c

(b) $AD = \dfrac{2bc}{b+c} \cos \dfrac{A}{2}$

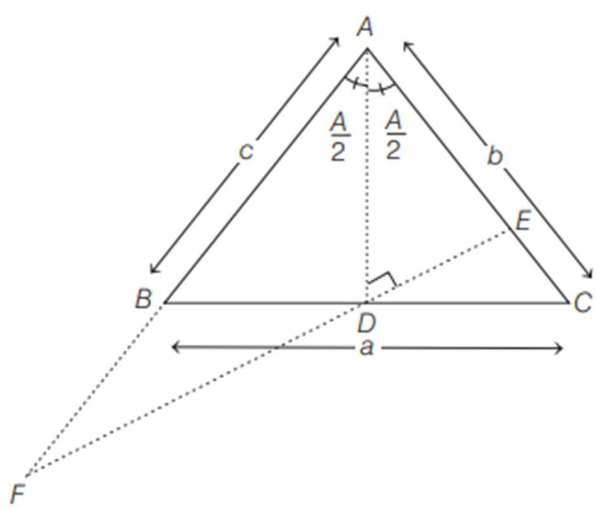
(c) $EF = \dfrac{4bc}{b+c} \sin \dfrac{A}{2}$

(d) ΔAEF is isosceles

Solution: -

Since, $\Delta ABC = \Delta ABD + \Delta ACD$

$$\Rightarrow \quad \frac{1}{2} bc \sin A = \frac{1}{2} c\, AD \sin \frac{A}{2} + \frac{1}{2} b\, AD \sin \frac{A}{2}$$



$$\Rightarrow \quad AD = \frac{2bc}{b+c} \cos \frac{A}{2}$$

Again, $AE = AD \sec \dfrac{A}{2} = \dfrac{2bc}{b+c}$

$\Rightarrow$ AE is HM of b and c.

$$EF = ED + DF = 2DE = 2AD \tan \frac{A}{2}$$

$$= 2 \frac{2bc}{b+c} \cos \frac{A}{2} \tan \frac{A}{2} = \frac{4bc}{b+c} \sin \frac{A}{2}$$

Since, $AD \perp EF$ and $DE = DF$ and AD is bisector.

$\Rightarrow$ ΔAEF is isosceles.

Hence, (a), (b), (c), (d) are correct answers.