Question: -

With the usual notation, in ΔABC, if $\angle A + \angle B = 120°$, $a = \sqrt{3} + 1$ and $b = \sqrt{3} - 1$, then the ratio $\angle A : \angle B$, is **(2019 Main, 10 Jan II)**

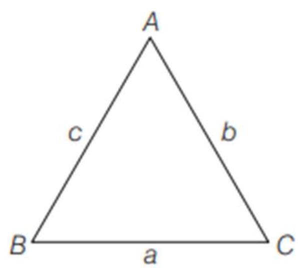
(a) $7:1$ (b) $3:1$

(c) $9:7$ (d) $5:3$

Solution: -

For a ΔABC, it is given that $a = \sqrt{3} + 1$,

$b = \sqrt{3} - 1$ and $\angle A + \angle B = 120°$



Clearly, $\angle C = 60°$ $[\because \angle A + \angle B + \angle C = 180°]$

Now, by tangent law, we have

$$\tan\frac{A-B}{2} = \frac{a-b}{a+b}\cot\frac{C}{2}$$

$$= \frac{(\sqrt{3}+1)-(\sqrt{3}-1)}{(\sqrt{3}+1)+(\sqrt{3}-1)}\cot\left(\frac{60°}{2}\right)$$

$$= \frac{2}{2\sqrt{3}}\cot(30°)$$

$$= \frac{1}{\sqrt{3}} \times \sqrt{3} = 1$$

$$\Rightarrow \tan\left(\frac{A-B}{2}\right) = 1 = \tan 45°$$

$$\Rightarrow \quad \frac{A-B}{2} = 45°$$

$$\Rightarrow \quad \angle A - \angle B = 90°$$

On solving $\angle A - \angle B = 90°$ and $\angle A + \angle B = 120°$, we get

$\angle A = 105°$ and $\angle B = 15°$

So, $\angle A : \angle B = 7:1$