Question: -

The set of all real numbers a such that $a^2 + 2a$, 2a + 3 and $a^2 + 3a + 8$ are the sides of a triangle is

(1985, 2M)

Solution: -

Since, $a^2 + 2a$, 2a + 3 and $a^2 + 3a + 8$ form sides of a triangle.

Now,
$$a^2 + 3a + 8 < (a^2 + 2a) + (2a + 3)$$

 $\Rightarrow a^2 + 3a + 8 < a^2 + 4a + 3$
 $\Rightarrow a > 5$...(i)
Also, $(a^2 + 3a + 8) + (2a + 3) > a^2 + 2a$
 $\Rightarrow 3a > -11$
 $\Rightarrow a > -\frac{11}{3}$...(ii)

Again,
$$(a^2 + 3a + 8) + (a^2 + 2a) > 2a + 3$$

 $\Rightarrow 2a^2 + 3a + 5 > 0$

which is always true.

 \therefore Triangle is formed, if a > 5