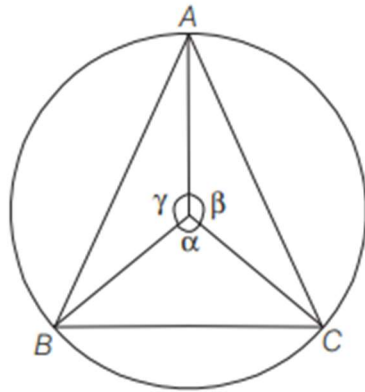


Question: -

The sides of a triangle inscribed in a given circle subtend angles α , β and γ at the centre. The minimum value of the arithmetic mean of $\cos\left(\alpha + \frac{\pi}{2}\right)$, $\cos\left(\beta + \frac{\pi}{2}\right)$ and $\cos\left(\gamma + \frac{\pi}{2}\right)$ is (1987, 2M)

Solution: -

Since, sides of a triangle subtends α, β, γ at the centre.



$$\therefore \alpha + \beta + \gamma = 2\pi \quad \dots(i)$$

Now, arithmetic mean

$$= \frac{\cos\left(\frac{\pi}{2} + \alpha\right) + \cos\left(\frac{\pi}{2} + \beta\right) + \cos\left(\frac{\pi}{2} + \gamma\right)}{3}$$

As we know that, $AM \geq GM$, i.e.

$$AM \text{ is minimum, when } \frac{\pi}{2} + \alpha = \frac{\pi}{2} + \beta = \frac{\pi}{2} + \gamma$$

$$\text{or } \alpha = \beta = \gamma = 120^\circ$$

\therefore Minimum value of arithmetic mean

$$= \cos\left(\frac{\pi}{2} + \alpha\right) = \cos(210^\circ) = -\frac{\sqrt{3}}{2}$$