Question: -

In a $\triangle ABC$, $a:b:c = 4:5:6$. The ratio of radius of the circumcircle to that of the incircle is… .   (1996, 1M)

Solution: -

We have, $R = \dfrac{abc}{4\Delta}$ and $r = \dfrac{\Delta}{s}$

$$\dfrac{R}{r} = \dfrac{abc}{4\Delta} \cdot \dfrac{s}{\Delta} = \dfrac{abc \cdot s}{4\Delta^2}$$

$$= \dfrac{abc}{4\,(s-a)\,(s-b)\,(s-c)}$$

But $a:b:c = 4:5:6$ [given]

$\Rightarrow$ $\dfrac{a}{4} = \dfrac{b}{5} = \dfrac{c}{6} = k$ [let]

$\Rightarrow$ $a = 4k,\ b = 5k, c = 6k$

Now, $s = \dfrac{1}{2}(a+b+c) = \dfrac{1}{2}(4k+5k+6x) = \dfrac{15k}{2}$

$\therefore \quad \dfrac{R}{r} = \dfrac{(4k)\,(5k)\,(6k)}{4\left(\dfrac{15k}{2} - 4k\right)\left(\dfrac{15k}{2} - 5k\right)\left(\dfrac{15k}{2} - 6k\right)}$

$$= \dfrac{30k^3}{k^3\left(\dfrac{15-8}{2}\right)\left(\dfrac{15-10}{2}\right)\left(\dfrac{15-12}{2}\right)} = \dfrac{30 \cdot 8}{7 \cdot 5 \cdot 3} = \dfrac{16}{7}$$