

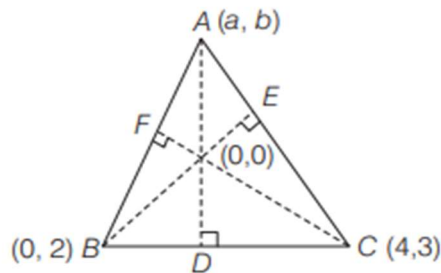
Question: -

Two vertices of a triangle are $(0, 2)$ and $(4, 3)$. If its orthocentre is at the origin, then its third vertex lies in which quadrant? (2019 Main, 10 Jan II)

- (a) Fourth (b) Third
(c) Second (d) First

Solution: -

Let ABC be a given triangle with vertices $B(0, 2)$, $C(4, 3)$ and let third vertex be $A(a, b)$



Also, let D, E and F are the foot of perpendiculars drawn from A, B and C respectively.

$$\text{Then, } AD \perp BC \Rightarrow \frac{b-0}{a-0} \times \frac{3-2}{4-0} = -1$$

[if two lines having slopes m_1 and m_2 , are perpendicular then $m_1 m_2 = -1$]

$$\Rightarrow b + 4a = 0 \quad \dots(i)$$

and $CF \perp AB$

$$\Rightarrow \frac{b-2}{a-0} \times \frac{3-0}{4-0} = -1$$

$$\Rightarrow 3b - 6 = -4a$$

$$\Rightarrow 4a + 3b = 6 \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$-b + 3b = 6 \Rightarrow 2b = 6$$

$$\Rightarrow b = 3$$

$$\text{and } a = -\frac{3}{4} \quad [\text{from Eq. (i)}]$$

So, the third vertex

$$(a, b) \equiv \left(-\frac{3}{4}, 3\right), \text{ which lies in II quadrant.}$$