Tricks :- [Applicable anywhere]

Definition :- The displacement of a particle through which it actually moves is called actual displacement of the particle.

Definition :- Dot product of applied force on the particle with actual displacement is called actual work done on the particle.

Definition :- Any imaginary displacement of a particle, which a particle doesn't follow, also maintaining the constraints (if any) is called the virtual displacement of the particle.

Definition :- Dot product of applied force on the particle with the virtual displacement is called the virtual work on the particle.

Theorem :- The net virtual work of all the forces of reaction/constraint (tension, normal reaction, static friction, etc.) is always 0 for any virtual displacement which is any harmony with the given Kinematic constraints.

Proof :- Out of scope.

Applications :-

It helps in finding constraint relations in any system with constraint forces.

Mathematical Representation of the Theorem

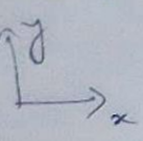
$$\sum_i \vec{F_i} \cdot \delta \vec{r_i} = 0 \qquad \text{--- (1)}$$

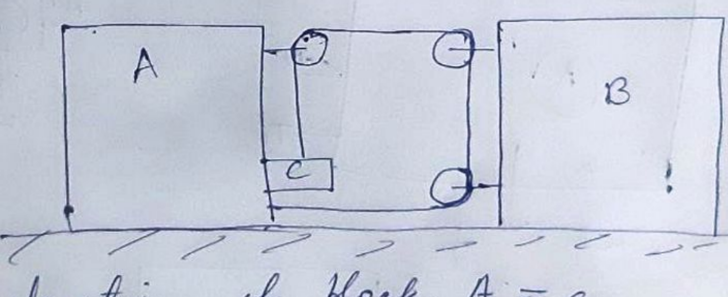
where $\vec{F_i}$ = ith constraint force

$\delta \vec{r_i}$ = ith virtual displacement in harmony with the constraint

Note that (1) holds $\forall\ \delta\vec{r_i} \in \mathbb{R}^3$.

Example :-





If acceleration of block $A = a$

acceleration of block $B = b$

Find the acceleration of C.

Soln :- Let acceleration of $C = a_x\hat{i} + a_y\hat{j}$

Now we use the above trick of virtual work to find $a_x, a_y \in \mathbb{R}$.

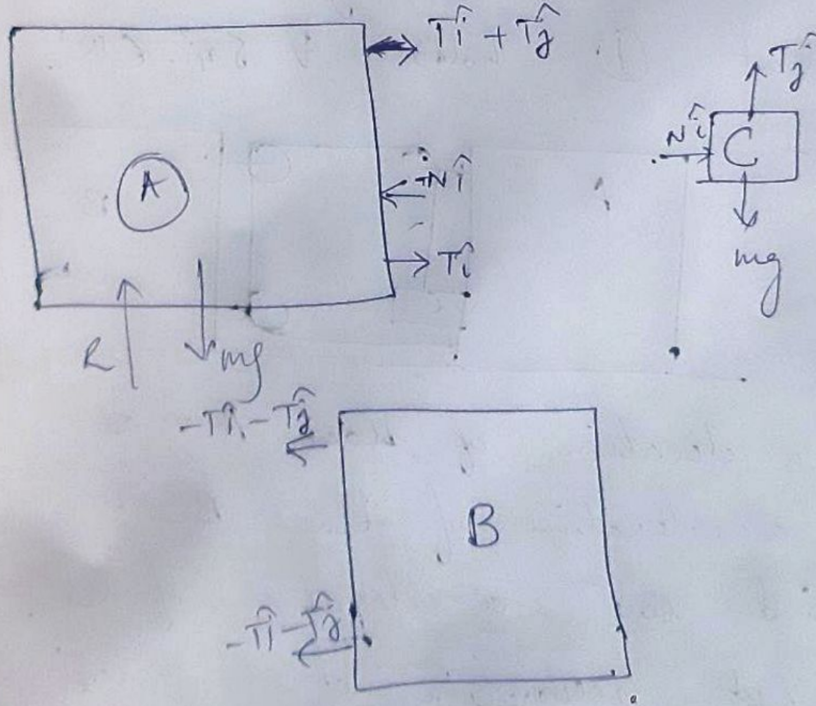
Since both objects A & B are constrained to move along x direction, we assume virtual displacements of A and B to be $x_A \hat{i}$ and $x_B \hat{i}$ respectively.

Note that $\frac{d^2 x_A}{dt^2} = a$, $\frac{d^2 x_B}{dt^2} = -b$

Since C is free to move in the plane we assume its virtual displacement to be $x_c \hat{i} + y_c \hat{j}$. Since $\vec{a_c} = a_x \hat{i} + a_y \hat{j}$

$$\Rightarrow \quad \frac{d^2 x_c}{dt^2} = a_x, \quad \frac{d^2 y_c}{dt^2} = a_y.$$

Drawing FBDs of A, B, C



Now,

According to ① $\Rightarrow \sum_i \vec{F_i} \cdot \delta \vec{r_i} = 0$

constrained)
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$= -b$
in the
placement
$\hat{i} + a_y \hat{j}$
$= 0$

$$\Rightarrow (T\hat{i}+T\hat{j})\cdot(x_A\hat{i}) + (T\hat{i})\cdot(x_A\hat{i}) + (-N\hat{i})\cdot(x_A\hat{i})$$
$$+ (N\hat{i})\cdot(x_c\hat{i}+y_c\hat{j}) + (T\hat{j})\cdot(x_c\hat{i}+y_c\hat{j})$$
$$+ (-T\hat{i}-T\hat{j})\cdot(x_B\hat{i}) + (-T\hat{i}-T\hat{j})\cdot(x_B\hat{i}) = 0$$

$$\Rightarrow Tx_A + Tx_A - Nx_A + Nx_c + Ty_c - Tx_B - Tx_B = 0$$

$[\because \hat{i}\cdot\hat{i} = 1 = \hat{j}\cdot\hat{j}$, $\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{i} = 0]$

$$\Rightarrow 2Tx_A - Nx_A + Nx_c + Ty_c - 2Tx_B = 0. \quad (2)$$

Since this eq^n holds $\forall\ x_A, y_c, x_B, x_c \in \mathbb{R}$ [According to theorem]

$\Rightarrow$ Coefficients of T and N are 0.

So (2)

$$\Rightarrow N(x_c - x_A) + T(2x_A + y_c - 2x_B) = 0$$

So equating the coefficients of N, T to be 0

$\Rightarrow x_c = x_A$

$\Rightarrow \frac{d^2x_c}{dt^2} = \frac{d^2x_A}{dt^2}$

$\Rightarrow \boxed{a_x = a}$

$\Rightarrow 2x_A + y_c - 2x_B = 0$

$\Rightarrow 2\frac{d^2x_A}{dt^2} + \frac{d^2y_c}{dt^2} - 2\frac{d^2x_B}{dt^2} = 0$

$\Rightarrow 2a + a_y + 2b = 0$

$\Rightarrow a_y = -2(a+b)$

$$\Rightarrow \boxed{\vec{a_c} = a\hat{i} - 2(a+b)\hat{j}}$$