

5) Consider the following system of equations:

$$\begin{aligned}x + 2y - 3z &= a \\ 2x + 6y - 11z &= b \\ x - 2y + 7z &= c\end{aligned}$$

where  $a, b$  and  $c$  are real constants. Then the system of equations:

- (a) has infinite numbers of solutions when  $5a = 2b + c$
- (b) has no solution for all  $a, b$  and  $c$
- (c) has a unique solution when  $5a = 2b + c$
- (d) has a unique solution for all  $a, b$  and  $c$ .

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Solution: (a)

$$\Delta = \begin{vmatrix} 1 & 2 & -3 \\ 2 & 6 & -11 \\ 1 & -2 & 7 \end{vmatrix} = 20 - 2(25) - 3(-10) = 0$$

$$\Delta_1 = \begin{vmatrix} a & 2 & -3 \\ b & 6 & -11 \\ c & -2 & 7 \end{vmatrix} = 4(5a - 2b - c)$$

$$\Delta_2 = \begin{vmatrix} 1 & a & -3 \\ 2 & b & -11 \\ 1 & c & 7 \end{vmatrix} = -5(5a - 2b - c)$$

$$\Delta_3 = \begin{vmatrix} 1 & 2 & a \\ 2 & 6 & b \\ 1 & -2 & c \end{vmatrix} = -2(5a - 2b - c)$$

For infinite solutions,  $\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

$$\rightarrow 5a = 2b + c$$